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The linear elastic analysis of cracked bodies, crack paths and some practical crack path examples

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ABSTRACT

The linear elastic analysis of cracked bodies is a Twentieth Century development, with the first papers appearing in 1907. The stress intensity factor concept, introduced in 1957, developed rapidly in the 1960s, with widespread application to practical problems. Finite element analysis had a significant influence on the development of linear elastic fracture mechanics. Corner point singularities were investigated in the late 1970s, but a new field parameter is still needed to describe the stresses at surfaces. There are geometric constraints on permissible fatigue crack paths. Some case studies of fatigue crack paths in metallic materials are included.

1. Introduction

The complete solution of a crack growth problem includes determination of the crack path. The first part of this paper is a brief review of the development of ideas on the linear elastic analysis of cracked bodies that are relevant to theoretical crack path determination. It is based on the author's personal involvement over more than 50 years. The second part of the paper is a collection of case studies on fatigue crack paths observed in metallic materials. Despite theoretical advances, fatigue crack paths are often determined by service simulation tests.

The review is restricted to linear elastic, homogeneous, isotropic materials, with any yielding confined to a small region at a crack tip. The first relevant papers had been published 50 years earlier, but in the late 1950s theoretical understanding of crack growth due to fatigue and static loadings was limited. The situation changed dramatically in the 1960s with the development of fracture mechanics, which is the applied mechanics of crack growth [1]. It was realised that linear elastic fracture mechanics, based on linear elastic analyses, sufficed for the solution of many practical engineering problems. By the mid 1970s practical applications of fracture mechanics were well established. In considering practical aspects of linear elastic fracture mechanics, scales of observation need to be taken into account since the scale chosen can make a considerable difference to the appearance of an object in general, and a crack in particular [2]. Scales of observation of 0.1 mm and above are usually described as macroscopic. The linear elastic concept of stress intensity factor describes the linear elastic stress field in the vicinity of a crack tip, and is a singularity. Stress intensity factors may be used to characterise the mechanical properties of cracked test pieces in just the same way that stresses are used to characterise the mechanical properties of uncracked test pieces. The conventional notation for the position of a point relative to the crack tip, and for the stresses at this point, is shown in Fig. 1. A point on the crack tip is the origin of the Cartesian coordinate system and the z axis lies along the crack tip. Displacements of points within the cracked body when the body is loaded are u, v, w in the x, y, z directions.

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Nomenclature

a	crack length, half crack length for an internal crack
B	biaxiality ratio
k	branch crack stress intensity factor, subscripts I, II denote mode
K	stress intensity factor, subscripts I, II, III denote mode, superscript c indicates a coupled mode
K_{max}	maximum value of K_I in fatigue cycle
K_λ	stress intensity measure, subscripts A, S denote mode
K_{Ic}	plane strain fracture toughness
MTS	maximum tensile stress
r	distance from crack tip
r, θ	polar coordinates
r, θ, ϕ	spherical coordinates
R	ratio of maximum to minimum load in fatigue cycle
T	T-stress
T_R	T-stress ratio
T_{Rc}	critical value of T_R
β	crack front intersection angle
β_c	critical crack front intersection angle
γ	crack surface intersection angle
ΔK	range of K_I in fatigue cycle
λ	parameter defining stress intensity measure
θ	initial crack growth direction
ν	Poisson's ratio
σ	stress, subscripts x, y, z denote direction
σ_Y	yield stress
τ	shear stress, subscripts xy, yz, xz denote direction

A fundamental fracture mechanics concept is that of crack tip surface displacement [3]. There are three possible modes of crack tip surface displacement, as shown in Fig. 2. These are: mode I where opposing crack surfaces move directly apart in directions parallel to the *y* axis; mode II where crack surfaces move over each other in the *xz* plane in directions parallel to the *x* axis, that is perpendicular to the crack tip; and mode III where crack surfaces move over each other in the *xz* plane in directions parallel to the *z* axis, that is parallel to the crack tip. By superimposing the three modes, it is possible to describe

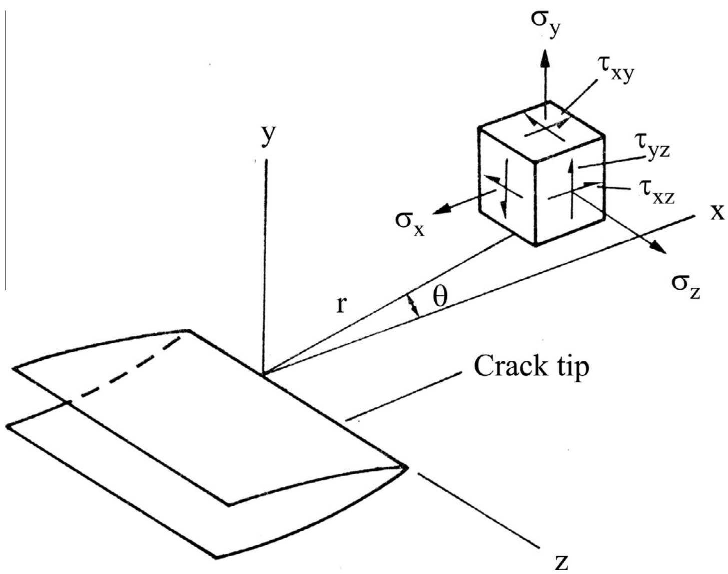

Fig. 1. Notation for crack tip stress field.