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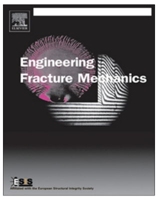

Fatigue and fracture assessment of notched components by means of the Strain Energy Density

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ABSTRACT

Brittle failure of components weakened by cracks or sharp and blunt V-notches is a topic of active and continuous research. It is attractive for all researchers who face the problem of fracture of materials under different loading conditions and deals with a large number of applications in different engineering fields, not only with the mechanical one. This topic is significant in all the cases where intrinsic defects of the material or geometrical discontinuities give rise to localised stress concentration which, in brittle materials, may generate a crack leading to catastrophic failure or to a shortening of the assessed structural life. Whereas cracks are viewed as unpleasant entities in most engineering materials, U- and V-notches of different acuities are sometimes deliberately introduced in design and manufacturing of structural components. Dealing with failure of notched components and summarising some recent experimental results reported in the literature, the main aim of the present contribution is to present a short review of the research work developed by Professor Paolo Lazzarin dealing with the Strain Energy Density approach. The approach, which is based on the volume Strain Energy Density (SED), has been recently applied to assess the failure of a large number of materials. The main features of the SED approach are outlined in the paper and its peculiarities and advantages accurately underlined. Some examples of applications are reported, as well.

1. Introduction

1.1. General overview

Dealing with fracture assessment of cracked and notched components a clear distinction should be done between large and small bodies [1]. The design rules applied to large bodies are based on the idea that local inhomogeneities, where material damage starts, can be averaged for large volume to surface ratio. In small bodies the high ratio between surface and volume makes not negligible the local discontinuities present in the material and the adoption of a multi-scaling and segmentation scheme is the only way to capture what happens at pico, nano and micro levels [1].

Keeping in mind the observations above and limiting our considerations to large bodies (i.e. large volume to surface ratio), for which an averaging process is still valid, the paper intents to review a volume-based Strain Energy Density approach applied to static and fatigue strength assessments of notched and welded structures [2–6].

The concept of “elementary” volume and “structural support length” was introduced many years ago [7] and it states that not the theoretical maximum notch stress is the static or fatigue strength-effective parameter in the case of pointed or sharp

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Nomenclature

E	Young's modulus, its plane strain value
e_1, e_2	coefficients for SED
K_{I}, K_{II}	conventional SIFs, mode I and II
K_1, K_2	NSIFs, mode 1 and 2
K_{IC}	critical SIF, mode I
K_t,	theoretical SCF
k	exponent in the S–N curve formula
q	factor related to notch opening angle
R_0	radius of control volume
r	radial distance
u_x, u_y	displacements in x, y directions
W, $\overline{W}$	SED, its mean value
W_c	critical mean SED value
x, y	Cartesian coordinates
α	notch opening semi-angle
θ	angular coordinate at slit tip
λ_1, λ_2	eigenvalues for NSIF, mode 1 and 2
v	Poisson's ratio
ρ	keyhole radius, notch root radius
σ_x, σ_y, τ_{xy}	stress components in Cartesian system
σ_θ, σ_r, $\tau_{r\theta}$	stress components in polar system
FE	finite element
FEM	finite element method
NSIF	notch stress intensity factor
SCF	stress concentration factor
SED	Strain Energy Density
SIF	stress intensity factor

notches, but rather the notch stress averaged over a short distance normal to the notch edge. In high cycle fatigue regime, the integration path should coincide with the early fatigue crack propagation path. A further idea was to determine the fatigue-effective notch stress directly (i.e. without notch stress averaging) by performing the notch stress analysis with a fictitiously enlarged notch radius, ρ_f, corresponding to the relevant support [8].

Dealing with notched components the idea that a quantity averaged over a finite size volume controls the stress state in the volume by means of a single parameter was first proposed by Sheppard [9].

For many years the Strain Energy Density (SED) has been used to formulate failure criteria for materials exhibiting both ductile and brittle behaviour. Since Beltrami [10] to nowadays the SED has been found being a powerful tool to assess the static and fatigue behaviour of notched and un-notched components in structural engineering. Different SED-based approaches were formulated by many researchers.

1.2. Provisional crack paths: a short review of the work by G. Sih

Dealing here with the strain energy density concept, it is worthwhile contemplating some fundamental contributions which are fundamental also for determining the provisional crack paths of engineering materials [11–17]. The concept of "core region" surrounding the crack tip was proposed in Ref. [11]. The main idea is that the continuum mechanics stops at a short distance from the crack tip, providing the concept of the radius of the core region. The strain energy density factor S was defined as the product of the strain energy density by a critical distance from the point of singularity [12]. Failure was thought of as controlled by a critical value S_c, whereas the direction of crack propagation was determined by imposing a minimum condition on S. The theory was extended to employ the total strain energy density near the notch tip, and the point of reference was chosen to be the location on the surface of the notch where the maximum tangential stress occurs. The strain energy density fracture criterion was refined and extensively summarised in Ref. [17]. The material element is always kept at a finite distance from the crack or the notch tip outside the "core region" where the in-homogeneity of the material due to micro-cracks, dislocations and grain boundaries precludes an accurate analytical solution. The theory can account for yielding and fracture and is applicable also to ductile materials. Depending on the local stress state, the radius of the core region may or may not coincide with the critical ligament r_c that corresponds to the onset of unstable crack extension. The ligament r_c depends on the fracture toughness K_{Ic}, the yield stress σ_y, the Poisson's ratio v and, finally, on the ratio between dilatational and distortional components of the strain energy density. The direction of $\sigma_{\max}$ determines maximum distortion while