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Electromechanical macroscopic instabilities in soft dielectric elastomer composites with periodic microstructures

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We study electromechanical macroscopic instabilities in dielectric elastomer (DE) composites undergoing finite strains in the presence of an electric field. We identify the unstable domains for DE composites with periodically distributed circular and elliptical inclusions embedded in a soft matrix. We analyze the influence of the applied electric field and finite strains, as well as the microstructure geometrical parameters and material properties, on the stability of the DE composites. We find that the unstable domains can be significantly tuned by an electric field, depending on the electric field direction relative to pre-stretch and microstructure. More specifically, the electric field aligned with the stretch direction, promotes instabilities in the composites, and the electric field applied perpendicularly to the stretch direction, stabilizes the composites. Critical stretch decreases with an increase in the volume fraction of circular inclusions. An increase in the contrast between the dielectric properties of the constituents, magnifies the role of the electric field, while an increase in the shear modulus contrast results in a less stable DE composite. For periodic DE composites with elliptical inclusions, we find that the critical stretch depends on the inclination angle of the inclusion, and that the critical stretch reaches a unique maximum at an angle defined by the inclusion ellipticity aspect ratio. In the aligned case – when the longest side of the inclusion is aligned with the stretch direction – an increase in the ellipticity ratio results in an increase in critical stretch.

1. Introduction

Dielectric elastomers (DEs) can achieve large deformations when excited by an electric field (Pelrine et al., 1998, 2000b, 2000a). This ability, together with their lightweight, fast response time and flexibility, make DEs attractive for a wide and diverse variety of applications, such as artificial muscles (Bar-Cohen, 2001), energy-harvesting and noise canceling devices, soft robotics (Kornbluh et al., 2012; McKay et al., 2010; Carpi et al., 2011; Bortot et al., 2016), and tunable waveguides (Gei et al., 2011; Galich and Rudykh, 2016). However, the wide spread usage of DEs has been limited due to the extremely high electric fields required to achieve large strains. Thus, DEs need to operate at the risky edge of electromechanical instabilities (Plante and Dubowsky, 2006; Rudykh et al., 2012; Keplinger et al., 2012; Li et al., 2013). A promising approach for reducing the required electric field is to design and fabricate composite materials with an enhanced electromechanical coupling. Experimental studies show significant enhancements in the electromechanical coupling in DE composites (Stoyanov et al., 2011; Huang and Zhang, 2004). Moreover, theoretical estimates and numerical simulations (Tian et al., 2012; Rudykh et al., 2013) predict even more significant improvements in the performance of DE composites with periodic microstructures. Thus, improvement by orders of magnitude in the electromechanical coupling can be achieved in hierarchically structured composites comprising softer and stiffer phases (Rudykh et al., 2013). Recent advances in the microstructured material fabrication and 3D printing, allowing realization of highly structured materials at different length-scales (Kolle et al., 2013; Lee and Fang, 2012; Zheng et al., 2014; Slesarenko and Rudykh, 2016), provide a great perspective for this approach for enhancing DE performance.

The foundation for the non-linear electroelasticity theory was laid by the pioneering works by Toupin (1956, 1960) showing that electromechanical coupling in DEs is characterized by a quadratic dependence on the applied electric field. Recently, the electroelasticity theory of finite deformations has been reformulated by Dorfmann and Ogden (2005, 2010), McMeeking and

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Landis (2005), and Suo et al. (2008); Zhao and Suo (2010), and, more recently, by Liu (2013), and by Li et al. (2016). Itskov and Khiêm (2014) and Ortigosa and Gil (2016) considered the aspects of the convexity of the electro-elastic energy functions. Cohen et al. (2016), Cohen and deBotton (2016) conducted a statistical-mechanics-based analysis of the response of polymer chain networks in DEs. In parallel, significant efforts have been made towards the development and implementation of the nonlinear electroelasticity framework into numerical schemes (Vu and Steinmann, 2007; Volokh, 2012; Javili et al., 2013; Keip et al., 2014; Galipeau et al., 2014; Jabareen, 2015; Aboudi, 2015). The electromechanical instabilities in finitely deformed homogenous DEs have been analyzed by Zhao and Suo (2007), and Dorfmann and Ogden (2010, 2014), in parallel with the experimental observations of the failure modes such as pull-in instabilities (Plante and Dubowsky, 2006), creasing and surface patterning (Wang et al., 2011). Based on an exact analytical solution available for finitely deformed periodic layered DE composites, the studies of the electromechanical instabilities in the periodic DE laminates have been performed (Bertoldi and Gei, 2011; Rudykh and deBotton, 2011; Rudykh and Bertoldi, 2013; Rudykh et al., 2014). These works show the significant dependence of DE material stability on the applied electric field and pre-stretch. However, the set of microstructures for which exact analytical solutions can be derived is limited; as a result, very little is known about the instabilities in DE composites with particulate and periodic microstructures, which showed promising results of significant enhancement in electromechanical coupling and actuation (Rudykh et al., 2013). Moreover, the knowledge about the instabilities in these microstructured electro-active composites may provide the tools for designing materials with switchable functionalities (Bertoldi et al., 2008; Bertoldi and Boyce, 2008; Krishnan and Johnson, 2009; Rudykh and Boyce, 2014; Singamaneni et al., 2008; 2009).

In this study, we perform an analysis of electromechanical instabilities in finitely deformed DE composites with periodically arranged active particles embedded in a matrix. In particular, we focus on the *macroscopic* stability of periodic two-dimensional DE composites with circular and elliptical inclusions. We implement the electromechanical instability analysis into a numerical finite element based tool, and identify the unstable domains for finitely deformed DE composites in the presence of an electric field. We analyze the influence of the electric field, pre-stretch, microstructure and material parameters on DE composite stability.

The work is structured as follows: Sec. 2 presents the theoretical background for the finitely deformed dielectric elastomers and electromechanical instability analysis previously developed by Dorfmann and Ogden (2005, 2010) and its specification for a plane problem reported in Rudykh et al. (2014). The numerical simulations, including the electromechanical periodic boundary conditions, and the procedure for determination of the electroelastic moduli are described in Sec. 3. In Sec. 4, we apply the stability analysis to identify the unstable domains for the DE composites with periodically distributed circular (4.1) and elliptical (4.2) inclusions embedded in a matrix. Sec. 5 concludes the paper with a summary and a discussion.

2. Theoretical background

We denote by $\mathscr{B}_0$ and $\mathscr{B}$ the regions occupied by a body in the reference and current configurations, respectively. The Cartesian position vector of a material point in the reference configuration of a body is $\mathbf{X}$ and its position vector in the deformed configuration is $\mathbf{x}$. We introduce a mapping vector function χ such that

$$\mathbf{x} = \chi(\mathbf{X}). \tag{1}$$

The deformation gradient is defined as

$$\mathbf{F} = \frac{\partial \chi(\mathbf{X})}{\partial \mathbf{X}}. \tag{2}$$

The ratio between the volumes in the current and reference configurations is $J \equiv \det\mathbf{F} > 0$.

We consider a quasi-static deformation in the absence of a magnetic field, electrical charges or electric currents within the material. Consequently, Maxwell equations take the form

$$\mathrm{Div}\mathbf{D}^0 = 0 \quad \text{and} \quad \mathrm{Curl}\mathbf{E}^0 = 0, \tag{3}$$

where $\mathbf{D}^0$ is the electric displacement and $\mathbf{E}^0$ is the electric field in the reference configuration. Note that Div$(\cdot)$ and Curl$(\cdot)$ are the differential operators in the reference configuration, while div$(\cdot)$ and curl$(\cdot)$ denote the corresponding differential operators in the current configuration. The referential electric field and electric displacement are related to their counterpart in the deformed configuration (Dorfmann and Ogden, 2005, 2010) via

$$\mathbf{E}^0 = \mathbf{F}^{\mathrm{T}}\mathbf{E} \quad \text{and} \quad \mathbf{D}^0 = J\mathbf{F}^{-1}\mathbf{D}. \tag{4}$$

We follow the analysis proposed by Dorfmann and Ogden (2005, 2010) and consider the elastic dielectrics whose constitutive relation is given in terms of a scalar-valued energy-density function $\Psi(\mathbf{F}, \mathbf{E}^0)$ such that

$$\mathbf{P} = \frac{\partial \Psi(\mathbf{F}, \mathbf{E}^0)}{\partial \mathbf{F}} \quad \text{and} \quad \mathbf{D}^0 = -\frac{\partial \Psi(\mathbf{F}, \mathbf{E}^0)}{\partial \mathbf{E}^0}, \tag{5}$$

where $\mathbf{P}$ is the total nominal stress tensor. The corresponding equations for an incompressible material are

$$\mathbf{P} = \frac{\partial \Psi(\mathbf{F}, \mathbf{E}^0)}{\partial \mathbf{F}} - p\mathbf{F}^{-T} \quad \text{and} \quad \mathbf{D}^0 = -\frac{\partial \Psi(\mathbf{F}, \mathbf{E}^0)}{\partial \mathbf{E}^0}, \tag{6}$$

where p is an unknown Lagrange multiplier. For an isotropic electroelastic material, an energy-density function Ψ can be expressed as a function of the six invariants

$$\Psi\left(\mathbf{F}, \mathbf{E}^0\right) = \Psi(I_1, I_2, I_3, I_{4e}, I_{5e}, I_{6e}), \tag{7}$$

where

$$I_1 = \mathrm{Tr}\mathbf{C}, \quad I_2 = \frac{1}{2}\left(I_1^2 - \mathrm{Tr}\mathbf{C}^2\right), \quad I_3 = \det\mathbf{C}, \tag{8}$$

$$I_{4e} = \mathbf{E}^0 \cdot \mathbf{E}^0, \quad I_{5e} = \mathbf{E}^0 \cdot \mathbf{C}^{-1}\mathbf{E}^0, \quad I_{6e} = \mathbf{E}^0 \cdot \mathbf{C}^{-2}\mathbf{E}^0, \tag{9}$$

where $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ is the right Cauchy-Green strain tensor. In the absence of body forces the equilibrium equation takes the form

$$\mathrm{Div}\mathbf{P} = 0. \tag{10}$$

The equilibrium equation in the *current* configuration is

$$\mathrm{div}\mathbf{T} = 0, \tag{11}$$

where the Cauchy stress tensor is related to the first Piola-Kirchhoff stress tensor via $\mathbf{T} = J^{-1}\mathbf{P}\mathbf{F}^T$.

Next we analyze small amplitude perturbations *superimposed* on the finitely deformation and electric field (Dorfmann and Ogden, 2010). The corresponding incremental equations are