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Propagation of guided elastic waves in nanoscale layered periodic piezoelectric composites

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ABSTRACT

This paper is concerned with the guided elastic waves propagating in nanoscale layered periodic piezoelectric composites. The equations of wave motion based on the nonlocal piezoelectricity continuum theory are derived, and the symmetric wave mode is considered. According to the continuity conditions of the mechanical and electric field quantities on the interface between the two neighboring sub-layers, we obtain the dispersion relation to analyze the behavior of the guided elastic waves and the influences of the nanoscale size-effect. A cut-off frequency appears when taking the nanoscale size-effect into consideration. The variations of the mechanical displacements and the electrical potential are calculated and discussed. The influences of the nanoscale size-effect and the volume fractions on the mode conversions are analyzed in details. It is found that all the dispersion curves including the mode conversion zones are compressed under the cut-off frequency. As the ratio of the internal to external characteristic lengths increases, the cut-off frequency decreases, while the frequency and the wave number of the mode conversion reduce. The present investigation may help us to control the cut-off frequency and the mode conversions by tuning the internal or external characteristic lengths and the volume fractions of the nanoscale layered periodic piezoelectric composites. The corresponding results may provide the theoretical basis for nanoscale wave device applications to control the wave mode conversions and the cut-off frequency.

1. Introduction

In the recent decades, many efforts have been devoted to the research on the wave propagation in layered piezoelectric structures. Early, Tiersten (1963) derived the solution of the wave propagation in an infinite piezoelectric plate using the linear piezoelectric theory. Exact solutions of the wave equations with the electromechanical coupling were obtained for some simple modes of two-dimensional (2D) waves in an infinite plate (Bleustein, 1969). Camley et al. (1983) demonstrated that a periodically layered semi-infinite medium parallel to the stress-free surface can support shear horizontal surface acoustic waves. The propagation of Lamb waves in a metallic semi-infinite medium covered with a piezoelectric layer was studied (Jin et al., 2002). Zhang and Geng (1994) obtained the variations of the mechanical displacement and electric fields in a piezo-composite with a 2-2 connectivity as well as the dispersion curves based on the theory of Lamb waves in a plate. Shui and Xue (1997) calculated the dispersion curves of Lamb waves propagating along the piezo-ceramic and polymer layers in the thickness direction of the 2-2 composite transducers. Pang et al. (2008) derived the dispersion relations of the wave propagation in layered periodic composites consisting of piezoelectric and piezomagnetic materials. They calculated the dispersion curves and the displacement fields, and analyzed the conversion of the longitudinal mode and the coupling mode. The dispersion behaviors and the band structures of the SH waves in a magnetic-electric (ME) periodically layered plate were investigated by Pang et al. (2014).

Since many novel device geometries require very small sizes, nanoscale piezoelectric materials and structures have been applied

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in communications and biomedical engineering, etc. Scientists have begun to pay attention to the superlattice structures, which was first put forward by Esaki and Tsu (1970). The dispersion curves of the bulk and surface acoustic waves in superlattices were computed by the transfer matrix method (Djafari-Rouhani et al., 1983; Nougaoui and Djafari-Rouhani, 1987). Some other methods to analyze the mechanical behaviors of the nanomaterials or nanoscale structures including the molecular dynamics simulations, higher-order continuum theories, such as the nonlocal elasticity (NLE) continuum theory (Eringen, 1983, 2006) and the surface/interface elasticity theory (Gurtin and Weissmuller, 1975; Gibbs, 1906) etc., have been also developed. Eringen (1983) developed the NLE theory and calculated the dispersion relations for Rayleigh surface waves, which shown a good agreement with the lattice dynamic calculations. Based on the NLE theory and Timoshenko beam theory, Ke et al. (2012, 2015) investigated the nonlinear vibration of piezoelectric nanobeams and the free vibration of nonlocal piezoelectric nanoplates. Based on the Eringen's nonlocal Kirchhoff plate theory, Jandaghian and Rahmani (2016) investigated the free vibration of functionally graded piezoelectric nanoplates under simply supported edge conditions. They studied the influences of the nonlocal parameter, various gradient indexes, mode numbers, aspect ratio and side-to-thickness ratio on the natural frequencies. Applying the transfer matrix method based on the NLE theory, Chen and Wang (2011) calculated the dispersion curves of anti-plane elastic waves propagating normally in a nanoscale layered periodic structure. The results were proved to be consistent with those calculated by the first principle (Ramprasad and Shi, 2005). After that, Chen et al. (2013) computed the band structures for the elastic waves propagating both normally and obliquely in a nanoscale periodic layered structure. A cut-off frequency, beyond which the elastic waves cannot propagate in the structure, was found when taking the nanoscale size-effect into account. Often, this cut-off frequency is also referred to as the escape frequency (Liu and Yang, 2012). The band structures for the anti-plane waves propagating in a nanoscale periodic layered piezoelectric structure was investigated (Chen et al., 2016).

Nevertheless, there are few research works on the propagation of the guided elastic waves in nanoscale periodic piezoelectric structures until now. In this paper, guided elastic waves propagating in the nanoscale layered periodic PVDF/PZT-5H structure are studied. The NLE continuum theory is extended to the nonlocal piezoelectric (NLPE) continuum theory by considering the electromechanical coupling effect. The corresponding governing equations of the wave motion are solved, the symmetrical wave mode is selected, and then the dispersion relations are derived. The variations of the mechanical displacements and the electric potential are presented and discussed to analyze the influences of the piezoelectricity and the nanoscale size-effect on the band structures and the wave mode conversions.

2. Problem statement and mathematical formulation

The layered periodic piezoelectric composites consisting of material A and material B alternately, together with the coordinate system, are displayed in Fig. 1. Both component materials are transversely isotropic piezoelectric solids. The *x*-*y* plane is the isotropic plane and the polarization direction is parallel to the *z*-axis. The *x*-axis is perpendicular to the interfaces between two neighboring layers, and the origin of the coordinate is at the center of the mid layer of material A. The volume fractions of the two materials A and B are $V^{(a)} = V$ and $V^{(b)} = 1$-V, respectively. Hereafter, the field quantities and the parameters of the constituents are denoted by the superscripts $(\cdot)^{(a)}$ and $(\cdot)^{(b)}$, respectively. Thus, the layers are characterized by their thicknesses $h^{(a)} = V^{(a)}h$ and $h^{(b)} = V^{(b)}h$ with $h = h^{(a)} + h^{(b)}$ being the thickness of one unit-cell, which are all at the nanoscale.

According to the NLPE continuum theory, the stresses and electric displacements at one point $\mathbf{x}$ in the body depend not only on the strains and electric fields at $\mathbf{x}$ but also on those at all other points of the whole body (Eringen, 2006). This observation on the phonon dispersion is in accordance with the atomic theory of lattice dynamics and experimental observations (Eringen, 2006). When the effects of the strains and electric fields at other points rather than $\mathbf{x}$ are neglected, the classical piezoelectricity (CPE) continuum theory is obtained. Thus, the relationships between the NLPE stress and electrical displacement components and the CPE components can be expressed as

$$\begin{aligned} \tau_{kl}(\mathbf{x}) &= \int_v \alpha(|\mathbf{x}' - \mathbf{x}|)\sigma_{kl}(\mathbf{x}')dv(\mathbf{x}'), \\ d_k(\mathbf{x}) &= \int_v \alpha(|\mathbf{x}' - \mathbf{x}|)D_k(\mathbf{x}')dv(\mathbf{x}'), \end{aligned} \tag{1}$$

where $\mathbf{x}$ is the position vector; τ_{kl} and d_k are the NLPE stress and electrical displacement components, respectively; σ_{kl} and D_k are the traditional stress and electrical displacement components, respectively; and$\alpha(|\mathbf{x}' - \mathbf{x}|)$ is the influence function which is dependent on the internal characteristic length ε, e.g., the atomic lattice constant. More detailed information about $\alpha(|\mathbf{x}' - \mathbf{x}|)$ can be found in Eringen (1983, 2006).

As shown in Fig. 1, the size effect can be neglected in the *z*-direction because the structure is infinite along this direction. Thus, the influence function $\alpha(|\mathbf{x}' - \mathbf{x}|)$ in this paper can be written as

$$\alpha(|\mathbf{x}' - \mathbf{x}|) = \delta(|z' - z|)\alpha(|x' - x|), \tag{2}$$

where $\delta(|x' - x|)$ is the Dirac-delta function. Consequently, the NLPE stress and electrical displacement components can be expressed by the CPE stress and electrical displacement components through

$$\begin{aligned} \tau^{(a)}_{kl}(\mathbf{x}) &= \int_v \alpha^{(a)}(|\mathbf{x}' - \mathbf{x}|)\sigma^{(a)}_{kl}(\mathbf{x}')dv(\mathbf{x}') = \int_{-\infty}^{+\infty}\int_{-h^{(a)}/2}^{h^{(a)}/2} \delta(|z' - z|)\alpha^{(a)}(|x' - x|)\sigma^{(a)}_{kl}(\mathbf{x}')dx'dz', \\ d^{(a)}_k(\mathbf{x}) &= \int_v \alpha^{(a)}(|\mathbf{x}' - \mathbf{x}|)D^{(a)}_k(\mathbf{x}')dv(\mathbf{x}') = \int_{-\infty}^{+\infty}\int_{-h^{(a)}/2}^{h^{(a)}/2} \delta(|z' - z|)\alpha^{(a)}(|x' - x|)D^{(a)}_k(\mathbf{x}')dx'dz', \end{aligned} \tag{3}$$