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Surface energy effect on nonlinear free vibration behavior of orthotropic piezoelectric cylindrical nano-shells

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ABSTRACT

In this paper, the electro-elastic surface/interface model is introduced to investigate the surface energy effect on the nonlinear free vibration behavior of orthotropic piezoelectric cylindrical nano-shells. On the basis of classical shell theory and von-Karman-Donnell-type geometric nonlinearity, the fundamental equations for vibration are given. By considering the constitute relations for surfaces, the total energy of the orthotropic piezoelectric cylindrical nano-shell is obtained. The governing equations of motion are derived from Hamilton's principle and solved by using the homotopy perturbation method (HPM). Afterwards, the results without surface effect are compared and validated with the datum available in the literature, and the influences of surface parameters and geometric characteristics on the nonlinear free vibration of the orthotropic piezoelectric cylindrical nano-shell are examined.

1. Introduction

Due to the outstanding piezoelectricity and the unique synergy between the piezoelectric and the semiconducting properties, piezoelectric materials have found a number of applications in smart structures and systems. Different from macroscopic piezoelectric objects, the piezoelectric nano-structures, such as nano-beams, nano-plates and nano-shells, possess not only the excellent piezoelectric effect but also the size-dependent mechanical and physical properties because of their large surface-to-volume ratio, which makes them very attractive in the nano-electro-mechanical systems (NEMS) (Lee et al., 2006; Li et al., 2007).

To better design the nano-shells and make them operate safely, the nonlinear vibration analysis of the nano-shells is quite important. Nano-structures are on such a small scale that the nano-sized effect is convinced to be significant, which has been demonstrated by both atomistic simulations and experimental investigations (Chen et al., 2006; Stan et al., 2007). Hence, the classical continuum theories can no longer be qualified to predict the size-dependent response of nano-structures.

In recent years, the nonlocal piezoelectric theory, surface/interface theory and others were proposed to predict the size-dependent response. In the nonlocal shell theory, the stress and electric displacement at a reference point depend on the strain components and electric-field components at the same position and those on all other points of the nano-structures. To consider the small scale effect, the scale coefficient related with a material parameter is introduced. Based on the nonlocal piezoelectric theory, the electro-thermal-mechanical buckling of double-walled Boron Nitride nanotubes was investigated, and the effects of electric and thermal loads on the critical buckling load were discussed (Ghorbanpour Arani et al., 2012).

In the surface/interface theory, the surface/interface is assumed to be very thin layers adhered to the adjacent media. Based on the first-order shear deformation theory as well as surface/interface elasticity theory raised by Gurtin and Murdoch (1975), the free vibration analysis of nano-shells was carried out and the effects of surface parameters on the free vibration behavior of nano-shells were presented (Rouhi et al., 2016). Nevertheless, the surface/interface piezoelectricity, as a unique character for piezoelectric material, is ignored in the surface elasticity theory, which makes it insufficient in predicting the size-dependent response of piezoelectric nano-structures. For example, Wang and Feng (2010) studied the vibration and buckling of piezoelectric nano-wires based on the surface elasticity theory. The effect of surface piezoelectricity, however, was neglected. It can be found that the surface piezoelectric effect may get considerable when the dimension of piezoelectric nano-structures comes into the nanoscale (Tagantsev, 1986). In recent years, to take into account the surface/interface piezoelectric effect, the electro-elastic couple surface/interface

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model (Fang et al., 2013; Liang et al., 2015) has been introduced to investigate the size-dependent effect on the behavior of piezoelectric nano-structures. By using this coupling surface/interface model, Fang et al. (2014) investigated the dynamic effective property of piezoelectric composites embedded with coated piezoelectric nano-fibers. Based on the Euler-Bernoulli beam theory combining with the electro-elastic surface/interface theory, Yan and Jiang (2011) investigated the surface effect on the electromechanical coupling and bending behaviors of piezoelectric nanowires.

For piezoelectric cylindrical nano-shells, besides the unique piezoelectricity, the mechanical-electrical coupling surface/interface effect gets considerable because of the size-dependent property, and the nonlinear behavior becomes more prominent when their vibration amplitude is comparable with the thickness of the nano-shell. By introducing the nonlinear higher order terms of strains, the nonlinear electro-thermo- mechanical buckling behavior of a piezoelectric cylindrical shell was studied, and the critical buckling load for the clamped supported mechanical and free electric potential boundary conditions at both ends of cylinder was given (Mosallaie Barzoki et al., 2013). However, to our best knowledge, no one has dealt with the nonlinear free vibration analysis of orthotropic piezoelectric cylindrical nano-shells based on the electro-elastic surface/interface theory.

The primary purpose of this paper is to present a further study on the nonlinear free vibration behavior of orthotropic piezoelectric cylindrical nano-shells based on the electro-elastic surface/interface theory. Hamilton's principle is employed to get the governing equations and an analytical solution is obtained by using the homotopy perturbation method (HPM). Then some numerical examples are given to illustrate the effect of surface elasticity, surface piezoelectricity and surface dielectricity on the nonlinear free vibration behavior of orthotropic piezoelectric cylindrical nano-shells.

2. Formulation

Fig. 1 depicts an orthotropic piezoelectric cylindrical nano-shell with the length L, thickness h, and mid-surface radius R. A curvilinear system, with its origin located on the mid-surface of the nano-shell, is adopted and the axial, circumferential and radial directions are described by x, θ and z, respectively. Due to the large ratio of surface to volume, the nano-shell is assumed to include a bulk part and two additional thin surfaces (the inner surface $S1$ and the outer surface $S2$), as shown in Fig. 1.

For the bulk part, the constitutive relations can be given as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{\theta\theta}\\ \sigma_{x\theta}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & 0\\ C_{21} & C_{22} & 0\\ 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \gamma_{x\theta}\end{Bmatrix}-\begin{bmatrix}0 & 0 & e_{31}\\ 0 & 0 & e_{32}\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_\theta\\ E_z\end{Bmatrix}$$
$$\text{or } \{\boldsymbol{\sigma}\}=[\boldsymbol{C}]\{\boldsymbol{\varepsilon}\}-[\boldsymbol{e}]\{\boldsymbol{E}\}, \tag{1}$$

$$\begin{Bmatrix}D_x\\ D_\theta\\ D_z\end{Bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ e_{31} & e_{32} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \gamma_{x\theta}\end{Bmatrix}+\begin{bmatrix}\eta_{11} & 0 & 0\\ 0 & \eta_{22} & 0\\ 0 & 0 & \eta_{33}\end{bmatrix}\begin{Bmatrix}E_x\\ E_\theta\\ E_z\end{Bmatrix}$$
$$\text{or } \{\boldsymbol{D}\}=[\boldsymbol{e}]^T\{\boldsymbol{\varepsilon}\}+[\boldsymbol{\eta}]\{\boldsymbol{E}\}, \tag{2}$$

where $\{\boldsymbol{\sigma}\}$, $\{\boldsymbol{\varepsilon}\}$, $\{\boldsymbol{E}\}$ and $\{\boldsymbol{D}\}$ respectively represent the vectors of stress, strain, electric field and electric displacement; $[\boldsymbol{C}]$, $[\boldsymbol{e}]$ and $[\boldsymbol{\eta}]$ respectively indicate the matrixes of elastic constant, piezoelectric

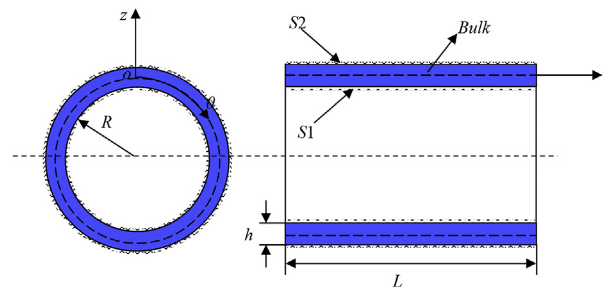

Fig. 1. Schematic view of the orthotropic piezoelectric cylindrical nano-shell.

constant and dielectric constant. Furthermore, the elastic constants can be written as

$$C_{11}=\frac{E_1}{1-\upsilon_{12}\upsilon_{21}},\ C_{12}=\frac{\upsilon_{21}E_1}{1-\upsilon_{12}\upsilon_{21}}=\frac{\upsilon_{12}E_2}{1-\upsilon_{12}\upsilon_{21}}=C_{21},\ C_{22}=\frac{E_2}{1-\upsilon_{12}\upsilon_{21}},\ C_{66}=G_{12}, \tag{3}$$

in which E_1 and E_2 are Young moduli along x and θ directions, respectively; G_{12} is shear modulus between the principal directions x and θ; υ_{12} and υ_{21} are Poisson ratios.

The components of electric field in the axial and circumferential directions can be expressed as a function of electric potential $\phi(x,\theta,t)$ as below (Ghorbanpour-Arani et al., 2013)

$$E_x=-\frac{\partial\phi}{\partial x},\ E_\theta=-\frac{1}{R}\frac{\partial\phi}{\partial\theta}, \tag{4}$$

For the inner and outer surfaces, based on the electro-elastic surface/interface theory, the constitutive relations can be introduced as follows (Fang et al., 2013)

$$\sigma_{ij}^S=\sigma_0^S\delta_{ij}+C_{ijkl}^S\varepsilon_{kl}^S-e_{kij}^SE_k^S, \tag{5}$$

$$D_i^S=e_{ikl}^S\varepsilon_{kl}^S+\eta_{ik}^SE_k^S, \tag{6}$$

where σ_0^S is the surface residual stress; C_{ijkl}^S, e_{kij}^S and η_{ik}^S are the elasticity tensor, piezoelectricity tensor and dielectricity tensor for the surfaces, respectively. In addition, the components of stress and electric displacement for the surfaces can be written as

$$\sigma_{xx}^{Sq}=\sigma_0^S+C_{11}^S\varepsilon_{xx}^{Sq}+C_{12}^S\varepsilon_{\theta\theta}^{Sq}-e_{31}^SE_x^{Sq}, \tag{7a}$$

$$\sigma_{\theta\theta}^{Sq}=\sigma_0^S+C_{21}^S\varepsilon_{xx}^{Sq}+C_{22}^S\varepsilon_{\theta\theta}^{Sq}-e_{32}^SE_\theta^{Sq}, \tag{7b}$$

$$\sigma_{x\theta}^{Sq}=C_{66}^S\gamma_{x\theta}^{Sq}, \tag{7c}$$

$$D_x^{Sq}=\eta_{11}^SE_x^{Sq}, \tag{8a}$$

$$D_\theta^{Sq}=\eta_{22}^SE_\theta^{Sq}, \tag{8b}$$

$$D_z^{Sq}=e_{31}^S\varepsilon_{xx}^{Sq}+e_{32}^S\varepsilon_{\theta\theta}^{Sq}+\eta_{33}^SE_z^{Sq}, \tag{8c}$$

in which the superscript q is equal to 1 and 2.

On the basis of the classical shell theory, the dynamic displacement field can be expressed as