On boundary conditions for buckling and vibration of nonlocal beams

C.M. Wang, H. Zhang, N. Challamel, W.H. Duan

PII: S0997-7538(16)30204-2

DOI: 10.1016/j.euromechsol.2016.08.014

Reference: EJMSOL 3344

To appear in: *European Journal of Mechanics / A Solids*

Received Date: 1 December 2015

Revised Date: 18 May 2016

Accepted Date: 31 August 2016

Please cite this article as: Wang, C.M., Zhang, H., Challamel, N., Duan, W.H., On boundary conditions for buckling and vibration of nonlocal beams, *European Journal of Mechanics / A Solids* (2016), doi: 10.1016/j.euromechsol.2016.08.014.

On boundary conditions for buckling and vibration of nonlocal beams

C. M. Wang[a], H. Zhang[b], N. Challamel[c] and W. H. Duan[d]

[a]Engineering Science Programme and Department of Civil and Environmental Engineering, National University of Singapore, Kent Ridge, Singapore 119260, E-mail: ceewcm@nus.edu.sg

[b]Department of Civil and Environmental Engineering, National University of Singapore, Kent Ridge, Singapore 119260, Corresponding author's E-mail: zhanghong@u.nus.edu

[c]Université Européenne de Bretagne, University of South Brittany UBS, UBS – LIMATB, Centre de Recherche, Rue de Saint Maudé, BP92116, 56321 Lorient cedex – France, E-mail: noel.challamel@univ-ubs.fr

[d]Department of Civil Engineering, Monash University, Clayton, Victoria 3800, Australia, E-mail: wenhui.duan@monash.edu

Abstract

In this paper, we argue that the boundary conditions for Eringen's nonlocal beam theory have to take on the discrete forms similar to those for the first order central finite difference beam model for beam buckling and vibration problems. The latter finite difference beam model has been found to be analogous to the physical Hencky bar-chain and therefore these two models can be regarded as one discrete beam model. Based on the phenomenological similarities between this discrete beam model and the Eringen's nonlocal beam theory, one may calibrate the Eringen's small length scale coefficient e_0 which is expected to be a constant appropriate to each material. When using the classical continuous nonlocal boundary conditions, we find that e_0 is not a constant but it depends on the cases of boundary conditions (such as pinned-pinned, clamped-clamped, clamped-pinned ends). However, this conundrum may be resolved if we use the discrete boundary conditions instead. By adopting the discrete boundary conditions, analytical solutions for e_0 may be obtained and the e_0 values finally converge to 0.289 for buckling problems and 0.408 for free vibration problems regardless of the cases of boundary conditions.

Keywords: nonlocal beam theory, Hencky bar-chain, boundary conditions

1. Introduction

Hencky (1920) proposed a discrete beam model comprising rigid beam segments (of equal length) connected by frictionless hinges and elastic rotational springs whose stiffness is given by $C = EI/a$ where EI is the flexural rigidity of the beam and a is the segmental length. Silverman (1951) pointed that the first order central finite difference beam model is analogous to the Hencky bar-chain since it can be shown that the discrete governing equations and boundary conditions are mathematically similar. Leckie and Lindberg (1963) and Wang et al. (2015a) showed that the central finite difference beam model and the Hencky bar-chain model are equivalent discrete beam models provided that the Hencky bar-chain model boundary conditions and intermediate restraint conditions take on spring stiffnesses as