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An edge dislocation near a nanosized circular inhomogeneity with interface slip and diffusion

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We study the transient elastic field induced by an edge dislocation near a nanosized circular elastic inhomogeneity in which the effects of interface slip and diffusion are incorporated into the model of deformation. Separate Gurtin-Murdoch surface elasticities are specified on the surface of the inhomogeneity and on the adjoining surface of the surrounding matrix. In addition, rate-dependent interface slip and diffusion are assumed to occur concurrently on the inhomogeneity-matrix interface. The ensuing interaction problem is solved using a simple yet effective method based on analytic continuation and a convenient decomposition of the proposed solution. In particular, our method allows us to circumvent the second-order tangential derivative taken with respect to the interfacial normal stress, typically a source of additional complication and often an obstacle to the solution of such problems. The original problem is reduced to two coupled linear algebraic equations and a number of mutually independent sets of state-space equations, the general solutions of which can be obtained by solving the associated generalized eigenvalue problem. The image force acting on the edge dislocation is derived using the Peach-Koehler formula. Corresponding stress and displacement fields as well as the image force are found to be dependent on four size-dependent dimensionless parameters (arising from the surface elasticities) and on two size-dependent parameters (having the dimension of time) arising from the incorporation of interface slip and diffusion and they evolve with an infinite number of size-dependent relaxation times.

1. Introduction

The interaction between dislocations and second-phase inhomogeneities plays an important role in the creep behaviors of composites and polycrystalline solids (Srolovitz et al., 1984; Wei et al., 2008). When the dimensions of the inhomogeneity lie in the nanometer range, the following two main factors take on major significance in the modelling and analysis of nanostructured solids. The first concerns the contribution of surface/interface stresses, tension and energies (Sharma and Ganti, 2004). These surface effects can be incorporated into continuum-based models by using the surface/interface model of Gurtin and Murdoch (Gurtin and Murdoch, 1975, 1978; Gurtin et al., 1998). The Gurtin-Murdoch surface elasticity model is equivalent to the assumption of a thin and stiff two-dimensional membrane perfectly bonded to the surface of a three-dimensional bulk (Steigmann and Ogden, 1997; Chen et al., 2007; Antipov and Schiavone, 2011; Markenscoff and Dundurs, 2014). The second major factor requiring consideration in nanostructured solids is rate-dependent interface slip and diffusion (Wei et al., 2008). Interface slip can be seen as local diffusion on a length scale comparable to the size of the asperities of the interface (Raj and Ashby, 1971), whilst long range interface diffusion is driven by the gradients of the chemical potential on the interface (Herring, 1950). Interface slip and diffusion contribute to room temperature plastic deformations in nanocrystalline materials. The co-existence of interface slip and diffusion makes the ensuing analysis extremely challenging even in the absence of surface elasticity (Wang and Pan, 2010, 2011).

This paper endeavors to consider the coupled effects of surface elasticity, interface slip, interface diffusion and dislocation emission/absorption on the transient deformations of nanostructured materials. More specifically, we investigate the elastic interaction between an edge dislocation and a nanosized circular

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inhomogeneity. The inhomogeneity and the matrix are first endowed with separate and distinct surface elasticities and are then bonded through an imperfect interface permitting rate-dependent slip and diffusion. An analytical solution in series form to this interaction problem is derived by means of complex variable methods. In contrast to the previous analysis in Wang and Pan (2010, 2011), we propose a simple yet effective method based on analytical continuation and a convenient decomposition of the solution which allows us to circumvent the involvement of the second-order tangential derivative (taken with respect to interfacial normal stress) normally appearing in the description of interface diffusion. This method has been adopted in a recent analysis of a nanosized circular inhomogeneity with interface slip and diffusion in the case when the surrounding matrix is subjected to uniform remote stresses (Wang et al., 2016). The time-dependent image force acting on the edge dislocation is then obtained using the acquired solution and the Peach-Koehler formula (Dundurs, 1969). We also present the asymptotic expression for the image force when the dislocation is located far from the inhomogeneity. Our analysis indicates that the stress and displacement fields in the composite as well as the normalized image force acting on the edge dislocation are dependent on four size-dependent dimensionless parameters $\gamma_1, \gamma_2, \delta_1, \delta_2$ arising from the surface elasticities and on two size-dependent parameters ρ, χ (having the dimension of time) arising from interface slip and diffusion and they evolve with an infinite number of relaxation times. These relaxation times rely, in turn, on the two size-dependent dimensionless parameters γ_1, γ_2 (arising from the surface elasticities) and on the two size-dependent parameters ρ, χ associated with interface slip and diffusion. Due to the existence of residual surface tensions, the normalized image force acting on a climbing dislocation, the Burgers vector of which is directed tangentially to the interface, is no longer an odd function of the dislocation position parameter and is actually dependent on a further size-dependent parameter which is given by the ratio of the radius of the inhomogeneity to the Burgers vector. We also present long range interaction results both in time and in space.

2. The coupled bulk-surface elasticity, interface slip and diffusion

In this section, the basic formulations describing the bulk elasticity, the surface elasticity and interface slip and diffusion are briefly summarized.

2.1. The bulk elasticity

In what follows, unless otherwise stated, Latin indices *i,j,k* take the values 1,2,3 and we sum over repeated indices. In the absence of body forces, the equilibrium equations and the stress-strain law describing the deformation of a linearly elastic, homogeneous and isotropic bulk solid can be expressed in a fixed rectangular coordinate system $\{x_i\}$ as follows

$$\sigma_{ij,j}=0, \quad \sigma_{ij}=2\mu\varepsilon_{ij}+\lambda\varepsilon_{kk}\delta_{ij}, \quad \varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right), \tag{1}$$

where λ and μ are Lame constants, σ_{ij} and ε_{ij} are, respectively, the Cartesian components of the stress and strain tensors in the bulk material, u_i is the *i*-th component of the displacement vector and δ_{ij} is the Kronecker delta.

For plane-strain problems, the stresses, displacements and stress functions φ_1, φ_2 can be expressed in terms of two analytic functions $\phi(z)$ and $\psi(z)$ of the complex variable $z=x_1+\mathrm{i}x_2$ as (Muskhelishvili, 1953; Ting, 1996)

$$\begin{aligned}
&\sigma_{11}+\sigma_{22}=2\left[\phi'(z)+\overline{\phi'(z)}\right],\\
&\sigma_{22}-\sigma_{11}+2\mathrm{i}\sigma_{12}=2\left[\bar{z}\phi''(z)+\psi'(z)\right],\\
&2\mu(u_1+\mathrm{i}u_2)=\kappa\phi(z)-z\overline{\phi'(z)}-\overline{\psi(z)},\\
&\varphi_1+\mathrm{i}\varphi_2=\mathrm{i}\left[\phi(z)+z\overline{\phi'(z)}+\overline{\psi(z)}\right],
\end{aligned} \tag{2}$$

where $\kappa=3-4\nu$ and ν $(0\le\nu\le 1/2)$ is Poisson's ratio. In addition, the stresses are related to the stress functions through (Ting, 1996)

$$\begin{aligned}
&\sigma_{11}=-\varphi_{1,2}, \quad \sigma_{12}=\varphi_{1,1},\\
&\sigma_{21}=-\varphi_{2,2}, \quad \sigma_{22}=\varphi_{2,1}.
\end{aligned} \tag{3}$$

Let T_1 and T_2 be traction components along the x_1-and x_2-directions on a boundary L. If s is the arc-length measured along L such that the material remains on the left-hand side in the direction of increasing s, it can be shown that (Ting, 1996)

$$T_1+\mathrm{i}T_2=-\frac{d(\varphi_1+\mathrm{i}\varphi_2)}{ds}. \tag{4}$$

2.2. The surface elasticity

The equilibrium conditions on the surface incorporating interface/surface elasticity can be expressed as (Gurtin and Murdoch, 1975, 1978; Gurtin et al., 1998; Ru, 2010)

$$\begin{aligned}
&\left[\sigma_{\alpha j}n_j\underline{e}_\alpha\right]+\sigma^s_{\alpha\beta,\beta}\underline{e}_\alpha=0, \quad \text{(tangential direction)}\\
&\left[\sigma_{ij}n_in_j\right]=\sigma^s_{\alpha\beta}\kappa_{\alpha\beta}, \qquad \text{(normal direction)}
\end{aligned} \tag{5}$$

where n_i are the components of the unit normal vector to the surface, [*] denotes the jump of the respective quantity across the surface, $\sigma^s_{\alpha\beta}$ are the Cartesian components of the surface stress tensor and $\kappa_{\alpha\beta}$ is the curvature tensor of the surface. In addition, the constitutive equations on the isotropic surface are given by

$$\sigma^s_{\alpha\beta}=\sigma_0\delta_{\alpha\beta}+2(\mu_s-\sigma_0)\varepsilon^s_{\alpha\beta}+(\lambda_s+\sigma_0)\varepsilon^s_{\gamma\gamma}\delta_{\alpha\beta}, \tag{6}$$

where $\varepsilon^s_{\alpha\beta}$ are the components of the surface strain tensor, σ_0 is the surface tension and λ_s and μ_s are the two surface Lame constants.

We mention that in Eqs. (5) and (6), the Greek indices α, β and γ take on values of the surface components. For example, in the case of circular cylindrical fibers, α, β, γ each take on the values θ, z.

2.3. Interface slip and diffusion

Let u_r and u_θ be the components of the displacement vector, normal and tangential, respectively, to the inhomogeneity-matrix interface L and σ_{rr},$\sigma_{r\theta}$ the normal and shear components, respectively, of the traction along the interface L. The interface slip and interface diffusion boundary conditions can then be explicitly expressed as (Koeller and Raj, 1978; Sofronis and McMeeking, 1994; Kim and McMeeking, 1995; Onaka et al., 1998; Wei et al., 2008)

$$[\sigma_{rr}+\mathrm{i}\sigma_{r\theta}]=0, \quad \sigma_{r\theta}=\vartheta[\dot{u}_\theta], \quad D\frac{d^2\sigma_{rr}}{ds^2}=-[\dot{u}_r], \quad \text{on } L, \tag{7}$$

where the overdot denotes differentiation with respect to time t, ϑ is the non-negative interface drag constant, D is the non-negative interface diffusion constant, and $[*]=[*]_M-[*]_I$ describes the jump across L. The definition of s in Eq. (7) is the same as that in Eq. (4). The interface slip is absent when $\vartheta\to\infty$; the interface diffusion is absent when $D=0$; the interface slip occurs much faster than the interface diffusion when $\vartheta=0$; the interface diffusion occurs much faster than the interface slip when $D\to\infty$. The appearance of the