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Nonlinear generalized thermoelasticity of an isotropic layer based on Lord-Shulman theory

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ABSTRACT

Thermoelastic analysis of an isotropic homogeneous layer within the framework of Lord-Shulman theory of generalized thermoelasticty is performed in this research. Two coupled partial differential equations, namely; the energy and equation of motion are established. The energy equation is kept in its original nonlinear form and the assumption made in previous investigations to linearize the energy equation is not established in the present work. The two coupled equations are presented in terms of axial displacement and temperature change. These equations are then transformed into a dimensionless presentation and discretised via the generalized differential quadrature method. The resulting equations are traced in time by means of the well-known $\beta-$ Newmark time marching scheme and solved iteratively at each time step. After validating the proposed approach and solution method for the case of thermally linear, a set of parametric studies are carried out to explore the effects of thermal shock magnitude, relaxation time, and the coupling parameter. It is shown that thermally nonlinear theory governs when thermal shock is severe, relaxation time is large, or coupling parameter is large.

1. Introduction

In the conventional Fourier law of heat transfer, temperature wave propagates with infinite speed which physically is in contradiction. To overcome such shortcoming, some non-classical theories are proposed. These theories are widely known as the generalized thermoelasticity theory, or thermoelasticity with second sound effect (Hetnarski and Ignaczak, 1999, 2000). Temperature wave, when is formulated within the framework of such theories, propagates with finite speed. In the most simple case, Lord and Shulman modified the conventional Fourier law by introducing a relaxation time and inserting the heat flux rate into the Fourier law (Hetnarski and Eslami, 2009). This theory results in finite speed of thermal wave propagation.

Layers and half-space are one-dimensional mediums with, in order, finite and infinite length in the Cartesian coordinates. Many researches are available on the response of layers and half-spaces subjected to thermal shock within the framework of coupled or generalized thermoelasticity. Hosseini zad et al (Hosseini Zad et al., 2012). applied the Galerkin finite element formulation to a unified formulation of generalized thermoelasticity of a layer or a composite layered media. Taheri et al (Taheri et al., 2004). obtained a solution for generalized thermoelasticity of a layer based on the Green-Naghdi theory. Bagri et al (Bagri et al., 2006). formulated the coupled and generalized thermoelasticity of a layer based on the various generalized thermoelasticity theories, namely the Lord Shulman, Green Lindsay, and the Green Naghdi. A unified formulation is developed which takes into account these three models (Hetnarski and Eslami, 2009). A closed form solution is obtained to study the waves propagation in time domain. Based on a unified formulation which consists of the classical Biot thermoelasticity, Lord-Shulman theory, Green-Naghdi theory, and the Green-Lindsay theory, coupled thermoelasticity of a half-space subjected to thermal shock is investigated by Youssef and El-Bary (Youssef and El-Bary, 2014). Wang et al (Wang et al., 2012). proposed a unified generalized thermoelasticity formulation which consists of three different theories, namely; the extended thermoelasticity, the temperature-rate-dependent thermoelasticity, and the thermoelasticity without energy dissipation. Laplace transform is used and the short time approximation is applied to obtain the displacements, temperature, and stresses in time domain. Hosseini-Tehrani and Eslami investigated the coupled thermoelasticity problem in a

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two-dimensional Cartesian domain based on various thermoelasticity models (Hosseini-Tehrani and Eslami, 2000a, 2003, 2000b). Boundary element method is applied successfully to the two-dimensional thermoelasticity equations to show that mechanical and thermal energy conversion takes place in a coupled field.

Youssef (Youssef, 2006) analysed the propagation of thermal waves in a two dimensional half-space medium made of an isotropic homogeneous material. Laplace and Fourier transformation techniques are used to solve the established equations. In this research, Lord-Shulman theory of generalized thermoelasticity is used to capture the finite speed of thermal wave propagation. Singh (Singh, 2011) developed a formulation for two dimensional generalized thermoelasticity in rectangular coordinates system which includes the effects of void and a single relaxation time. It is concluded that four coupled longitudinal waves and a shear wave are observed. He et al (He et al., 2015). proposed a solution method based on the Laplace transformation technique to demonstrate the propagation of temperature and displacement waves in a two dimensional half space. A memory dependent thermoelasticity formulation is provided by Yu et al (Yu et al., 2014). which contains a Kernel function and an extra time. This theory contains the Lord-Shulman theory as an especial case. Sarkar (Sarkar, 2013) presented the fundamental equations of the problem of generalized thermoelasticity with one relaxation time parameter including heat sources in the form of a vector-matrix differential equation in the Laplace transform domain and then solved them by the eigenvalue approach. Othman et al (Othman et al., 2009). presented the two dimensional generalized thermoelasticity formulation based on the Green-Naghdi type II and III assumptions for a half-space subjected to rapid heating. A state space formulation is presented by Youssef et al (Youssef and Al-Lehaibi, 2007). to deal with the thermoelasticity solution of an unbounded one dimensional medium within the context of two temperature generalized thermoplasticity.

As the above literature survey and further examination of other available works reveal, wealth investigations are available on the coupled and generalized thermoelasticity in one and two dimensional medium in the Cartesian domains of finite or infinite lengths. However, in all these researches, the first law of thermodynamics is linearised and, in coupling terms, the temperature change is ignored in comparison with the reference temperature. Such assumption is valid only when temperature change is much smaller than the reference temperature. This study is focused on the case when the thermal shock results in severe temperature changes which are comparable with the reference temperature. The governing equations are obtained for a layer under the assumption of single relaxation time consistent with the Lord-Shulman theory. The resulting equations are converted into a dimensionless form. The coupled partial differential equations are discretised via the generalized differential quadrature (GDQ) method in axial direction and integrated in time via the $\beta-$ Newmark time marching technique. Numerical results are provided for both thermally linear and nonlinear cases of first law of thermodynamics. As expected, the linearisation assumption fails when the applied thermal shock is severe.

2. Equation of motion

Consider an isotropic homogeneous layer of finite length a. In the absence of body forces, the equation of motion of the layer takes the form (Hetnarski and Eslami, 2009)

$$\frac{\partial \sigma_{xx}}{\partial x} = \rho \ddot{u} \tag{1}$$

where for an isotropic homogeneous elastic body the only present component of stress field, axial stress, is written in terms of the axial strain as (Hosseini Zad et al., 2012)

$$\sigma_{xx} = (2\mu + \lambda)\varepsilon_{xx} - \beta(T - T_0) \tag{2}$$

In this equation, λ and μ are the Lamé constants and β is thermoelastic parameter. Thermoelastic parameter may be obtained in terms of the Lamé constants and thermal expansion coefficient as $\beta = (3\lambda + 2\mu)\alpha$. Furthermore, T_0 is reference temperature and T is the absolute temperature within the body of the layer.

The axial component of strain in Cartesian coordinates takes the form

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \tag{3}$$

The axial equation of motion in terms of axial displacement is obtained when Eqs. (2) and (3) are inserted into Eq. (1). This equation is simply obtained as

$$(2\mu + \lambda)\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial T}{\partial x} = \rho \ddot{u} \tag{4}$$

3. Energy equation

Lord and Shulman modified the conventional Fourier law of heat conduction by introducing a relaxation time. The relaxation time represents the time-lag needed to establish steady state heat conduction in a volume element when a temperature gradient is suddenly imposed on the element. According to the Lord-Shulman theory, the conventional Fourier diffusion law is modified by introduction of a relation time t_0 as follows (Hetnarski and Eslami, 2009)

$$q + t_0\dot{q} = -K\frac{\partial T}{\partial x} \tag{5}$$

where q is the heat flux and K is thermal conductivity. For the case of $t_0 = 0$, the above equation simplifies to the classical Fourier law. The heat balance for an element of a body relating the axial heat flux q to the rate of specific heat influx Q is (Noda et al., 2003)

$$\dot{Q} = -\frac{\partial q}{\partial x} \tag{6}$$

and from the second law of thermodynamics

$$\delta Q = TdS \tag{7}$$

The above equation, when written in rate form, takes the form

$$\dot{Q} = T\dot{S} = T\left(\frac{\partial S}{\partial \varepsilon_{xx}}\dot{\varepsilon}_{xx} + \frac{\partial S}{\partial \theta}\dot{\theta}\right) \tag{8}$$

which simplifies to

$$\dot{Q} = T\beta\dot{\varepsilon}_{xx} + c_\varepsilon \rho \dot{T} \tag{9}$$

Equations (5), (6) and (9) are combined together to obtain the thermally nonlinear first law of thermodynamics in a layer based on the Lord-Shulman theory