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An analytical solution for the mechanochemical growth of an elliptical hole in an elastic plane under a uniform remote load

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ABSTRACT

A closed-form solution is presented for the mechanochemical corrosion of an elastic infinite plane containing an elliptical hole and subjected to uniform remote tension or compression. The corrosion is defined as the dissolution of the material along the surface of the hole with a rate which is proportional to the maximum principal stress at the corresponding points on the surface. Moreover, the dissolution rate can decay exponentially with time. It is confirmed that there exists a threshold load, such that the hole's shape elongates/evolves towards a circle when the applied stress lies above/below this threshold in absolute value. This threshold is determined by the relationship between the general corrosion of unstressed material and corrosion sensitivity to stresses. Formulas for the assessment of the lifetime of a plane in the case of the stress growth are obtained with inhibition of corrosion being taken into account.

1. Introduction

At the interface between a solid and a chemically active environment, the solid may lose its mass (corrode or dissolve). Common in many engineering applications are stress-assisted chemical reactions which may lead to premature failure of solids and structures. For different types of corrosion there have been proposed a number of relationships between the rate of propagation of the front of the chemical reaction and the effective stress: among them are linear, quadratic, cubic, and various kinds of exponential dependencies (Charles and Hillig, 1961; Dolinskii, 1967; Elishakoff et al., 2012; Gutman, 1994; Liang and Suo, 2001; Pavlov et al., 1985; Petrov et al., 1987; Rusanov, 2007).

The loss of material changes the solid's geometry and consequently its stress state, which, in turn, affects the reaction rate. In such situations we have to solve initial boundary value problems with unknown moving boundaries. Due to the complexity of the algorithms, such problems are often investigated numerically; but there are a number of analytical solutions, e.g. (Dolinskii, 1967; Elishakoff et al., 2012; Gutman et al., 2015; Karpunin et al., 1975; Petrov et al., 1987; Rusanov, 2007; Sedova and Pronina, 2015a). However, these papers deal with the corrosion kinetics when the effective stress (affecting the chemical reaction) is uniform along the whole interface.

Non-equal-rate stress assisted interfacial reactions have been studied by Dolinskii (1975); Petrov et al. (1987); Prevost et al. (2001); Srolovitz (1989); Tang et al. (2002) and others (e.g., see reviews in (Liang and Suo, 2001; Tang and Li, 2007)) by the use of perturbation analysis, variational methods and different numerical techniques. One of the problems of practical importance is the problem of stress distribution around various shaped holes in plates under different loading (Bashkankova et al., 2015; Jafari et al., 2016; Weissgraeber et al., 2016). The evolution of an initially elliptical cavity was simulated by Charles and Hillig (1961); Chuang and Fuller (1992); Tang et al. (2002) on the basis of a kinetic law where the stress can affect both the driving force and the activation energy of the reaction. Charles and Hillig (1961) considered an elliptical-shaped cavity embedded in a solid subjected to a remote tensile load applied in a direction perpendicular to its major axis under the assumption that the geometry of the cavity retained an elliptical shape during chemical dissolution. Chuang and Fuller (1992) used this model to identify different regimes of crack tip geometry under uniform remote stress. Their models proved the existence of threshold load (defined as a static fatigue limit) that discriminates between blunting and sharpening of the elliptical flaw. These advanced models developed in the framework of partial-equilibrium thermodynamics are widely applied in the numerical simulation of crack propagation, mainly for “ideal examples” such as glasses, ceramic materials, and amorphous metals (Liang and Suo, 2001; Tang and Li, 2007).

Unfortunately, the mechanisms of stress-enhanced corrosion

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are still not completely understood, especially for polycrystalline metals possessing a complex microstructure. In engineering practice the linear empirical relationship between the corrosion rate and stress is widely used (Dolinskii, 1967, 1975; Fridman and Elishakoff, 2015; Gutman et al., 2000; Karpunin et al., 1975; Petrov et al., 1987; Sedova and Pronina, 2015b). This linear dependence can also be considered as an approximation of an exponential or some other function for the rate of a material dissolution (the first two terms in the expansion of the function into Taylor's series in terms of mechanical stress) (Fridman, 2014).

The present paper concerns the problem of mechanochemical corrosion around an elliptical hole in an infinite plane under uniform remote tension, taking into account the possible inhibition of corrosion. We pay no attention to the thermodynamic details but consider an approximate analytical approach using the linear empirical law of corrosion kinetics to provide closed form solutions that can be more easily exploited (rather than performing complex numerical calculations). Previously obtained formulas (e.g., Pronina (2011, 2013, 2015)) for the mechanochemical dissolution of thick-walled cylinders and spheres can be employed to the problems of a large enough solid containing a cylindrical or spherical cavity. However, those solutions are not suitable for modelling a non-equal-rate growth of the cavity. In the present paper, analytical solutions are developed for the non-uniform growth of a hole under the assumption that the hole remains elliptical during the corrosion process, as was assumed in (Charles and Hillig, 1961).

It is worth noting that the term “mechanochemical corrosion” was introduced by Gutman (1994) for general corrosion enhanced by stress and it was not supposed to be used for local or fatigue corrosion. The empirical corrosion kinetic law by Dolinskii (1967), used in this article, is applicable to the general corrosion of metals as well. When the surface of a hole is the only surface completely exposed to dissolution and smooth enough, this term seams to be applicable. Accordingly, we consider the general corrosion around a sufficiently large hole until a localization of the corrosion process (if it happens).

2. Problem formulation

Consider a homogeneous, isotropic, and linearly elastic infinite plane S with an elliptical hole bounded by the contour L. The plane is supposed to be subjected to remote uniform tension or compression $\sigma^{\infty} = q$. The cavity surface is stress free and exposed to mechanochemical corrosion defined as material dissolution. It is assumed that the curvature of the cavity surface at every point is small enough and can not change its macro-electrochemical homogeneity and affect the corrosion process in unstressed material. In this situation the hole grows with time and the contour L changes with time as well. Let t be a variable representing the time elapsed from some reference time $t = t_0$. Let A_0 and B_0 be the semi-axes of the ellipse L at $t = t_0$. According to (Dolinskii, 1967, and others), the rate of corrosion, v, is a linear function of the stress value at the corresponding points s on the surface:

$$v(s) = \frac{d\delta(s)}{dt} = [a + m\,\sigma_1(s)]\exp(-bt), \quad s \in L(t). \tag{1}$$

Here, a, m, and b are empirically determined constants of the corrosion kinetics, $d\delta$ is the increment of the depth of material dissolution (during the time interval dt) at a position s on the hole's surface in the direction of its normal, σ_1 is the maximum principal stress. Note that the sign of m is the same as the sign of σ_1. The exponential factor in (1) describes the possible inhibition of corrosion (Pavlov et al., 1987). In general, the corrosion kinetics constants are different for tension and compression.

It is required to track the change in the stress concentration factor and the hole sizes with time.

3. Problem solution

3.1. Accepted assumption

Let us assume that the geometry of the hole retains an elliptical shape during the corrosion process. This assumption will reduce the complexity of the mathematical analysis and permit constructing an analytical solution for the growth of the axes of the hole. What actually happens everywhere along the reactant surface except at its vertices is neglected.

3.2. Basic equations

The first fundamental problem of a linearly elastic, isotropic infinite plane with an elliptical hole was solved by Muskhelishvili (1954) using conformal transformations. The area S external to the contour L in the z-plane (with the origin at the centre of the elliptical hole) is mapped conformally to the area outside the unit circle in the ζ-plane, $|\zeta|>1$. The mapping function is (Muskhelishvili, 1954)

$$z = R(\zeta + M/\zeta), \quad R > 0, \quad 0 \le M < 1,$$

where $\zeta = \rho e^{i\theta}$; $z = x+iy$; the axes $0x$ and $0y$ coincide with the semi-axes A and B of the elliptical hole. Moreover, $A = R(1+M)$ and $B = R(1-M)$.

Then the principal stresses along the hole surface in the plane under uniform remote tension or compression $\sigma_{xx}^{\infty} = \sigma_{yy}^{\infty} = q$ are

$$\sigma_{\theta\theta}(\theta) = 2q\frac{1 - M^2}{1 - 2M\cos 2\theta + M^2}, \quad \sigma_{\rho\rho}(\theta) = 0. \tag{2}$$

In the framework of the assumption made in Section 3.1, the stress components along the edge of the hole at any time are determined by the instantaneous values of the semi-axes A and B of the ellipse $L(t)$. Then, for the strength analysis, we need only track the growth of these semi-axes, A and B. According to (1), the rate of their growth depends on the maximum principal stress at the related points on the contour. Let us denote these stresses by σ^A and σ^B, so that $\sigma^A = \sigma_{\theta\theta}|_{z=\pm A}$ and $\sigma^B = \sigma_{\theta\theta}|_{z=\pm iB}$. Then it follows from (2) that

$$\sigma^A = \sigma_{\theta\theta}(0) = \sigma_{\theta\theta}(\pi) = 2q\,A/B, \quad \sigma^B = \sigma_{\theta\theta}(\pm\pi/2) = 2q\,B/A. \tag{3}$$

Substituting σ^A and σ^B defined by (3) for $\sigma_1(s)$ into (1) at $s = \pm A$ and $s = \pm iB$, we obtain the system of ordinary differential equations

$$\frac{dA}{dt} = \left[a + 2mq\frac{A}{B}\right]\exp(-bt), \tag{4}$$

$$\frac{dB}{dt} = \left[a + 2mq\frac{B}{A}\right]\exp(-bt). \tag{5}$$

The initial conditions to be satisfied at $t = t_0$ are

$$A(t_0) = A_0, \quad B(t_0) = B_0. \tag{6}$$

As can be seen from (3), on solving this system, the stress concentration factor can easily be calculated as

$$K = \max|\sigma_{\theta\theta}|/|q| = 2A/B = 2\eta, \quad (\eta \ge 1), \tag{7}$$

where the ratio between the axes,