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Bounds for the overall properties of composites with time-dependent constitutive law☆

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ABSTRACT

This paper deals with the search, via variational methods, of bounds on the overall mechanical properties of composite materials, with the constitutive laws of the constituents governed by linear operators, generally non-symmetric with respect to the chosen bilinear form. For these types of problems, by virtue of a symmetrization technique derived by Tonti (1984), we provide a minimum formulation, then used to derive bounds for the overall properties of composites having a linear time-dependent constitutive law. Some of the examples already known in the literature prove to be special cases of the theory proposed here, such as the results derived by Cherkaev and Gibiansky (1994) and Milton (1990), those obtained by Rafalski (1969a, 1969b) and Reiss and Haug (1978), and those provided by Carini and Mattei (2015).

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1. Introduction

The aim of this paper is to determine bounds for the overall mechanical properties of composites with phases having linear time-dependent constitutive laws.

So far, most of the theoretical developments in the homogenization field have been carried out for the specific case of elastic composites, for which the classical energetic theorems can be used. In particular, we recall the Voigt (1889) and Reuss (1929) bounds, and the Hashin and Shtrikman (1962, 1963) bounds.

On the other hand, very few results have been obtained for composites with constitutive laws ruled by non-potential operators, that is non-symmetric operators with respect to a bilinear form of the classical type. For the linear viscoelastic case we recollect the works by Christensen (1968) and Huet (1995).

The most significant work seems to be the one by Cherkaev and Gibiansky (1994), who provided a procedure to obtain extremum formulations for the conductivity problem and, above all, for the linear viscoelastic problem, in case the constitutive law operator be expressed in terms of complex moduli. In particular, considering the latter case, the procedure is based on the separation of the constitutive law operator into its real and complex parts, and on the application of a partial Legendre transform to the new split constitutive law. Milton (1990) extended Cherkaev and Gibiansky's method to problems ruled by non symmetric operators, by combining the given problem with its adjoint and by applying a partial Legendre transform to the new constitutive law.

Recently, for the linear viscoelastic problem, a new extremum formulation has been obtained by Carini and Mattei (2015). The latter is based on the division of the time domain into two equal subintervals with the consequent splitting of the equations of the problem. In particular, the constitutive law operator is split into sub-operators and it can be written as a two-by-two matrix, symmetric but not positive definite with respect to a time convolutive bilinear form. Applying a partial Legendre transform, as in the procedure conceived by Cherkaev and Gibiansky (1994), Carini and Mattei reformulated the problem so that the associated quadratic form turns out to be convex. We point out that very few minimum formulations have been presented in the time domain for a finite time interval. In fact, other formulations valid on an infinite time interval were proposed, for instance, by Rafalski (1969a,b), and Reiss and Haug (1978).

In this paper, using some ideas of Tonti (1984), we provide a method to symmetrize any type of linear constitutive law, for the purpose of obtaining bounds of the overall properties of linear composites. In particular, the results proposed by Cherkaev and Gibiansky (1994) and Milton (1990), Rafalski (1969a,b) and Reiss and Haug (1978), and Carini and Mattei (2015), obtained using very different methods, prove to be particular cases of the approach presented here.

In Section 2 we present an overview of the main results, derived following the procedure presented in Section 3. The details of the application of the method are illustrated in Section 4, while in

☆ In memory of Professor Osvaldo De Donato.

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Section 5 the concluding remarks are presented.

2. Summary of the results

We focus our attention on Solid Mechanics problems related to random composite media, under the hypothesis of small displacements and strains. In particular, we consider the problem on the Representative Volume Element (RVE) of a composite material with a time-dependent constitutive law. The aim is to provide bounds on the overall mechanical behavior of the composite, for every moment of time t in the interval $T=[0,\mathscr{T}]$, with $\mathscr{T}>0$, the solid being undisturbed for $t<0$.

A word about the notation may be helpful. We adopt lightface letters to indicate scalars and boldface letters to denote vectors, second-order and fourth-order tensors. In particular, fourth-order tensors are indicated by capital Latin letters while lower case Greek and Latin letters are used to indicate second-order tensors and vectors, respectively. Double-struck symbols represent matrices or vectors having vectors, fourth- and second-order tensors as components. For vectors, second-order and fourth-order tensors a symbolic simplified notation consisting in the mere juxtaposition of the respective symbols with no explicit tensorial subscripts is used. For example, $\mathbf{A}\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}\mathbf{A}$ indicate the second order inner products $A_{ijhk}\varepsilon_{hk}$ and $\varepsilon_{ij}A_{ijhk}$, respectively, whereas $\boldsymbol{\varepsilon}\mathbf{A}\boldsymbol{\varepsilon}$ denotes the quadratic form $\varepsilon_{ij}A_{ijhk}\varepsilon_{hk}$. Furthermore, $\boldsymbol{\varepsilon}\mathbf{x}$ and $\mathbf{x}\boldsymbol{\varepsilon}$ indicate the once contracted tensor product $\varepsilon_{ij}x_j$ and $x_i\varepsilon_{ij}$, respectively, whereas $\boldsymbol{\varepsilon}\boldsymbol{\sigma}$ denotes the twice contracted tensor product $\varepsilon_{ij}\sigma_{ij}$. Finally, $\mathbf{u}\,\mathbf{v}$ indicates u_iv_i. The indicial notation will be adopted only if strictly necessary.

Let us denote by V the volume of the region Ω occupied by the RVE, and by Γ the external surface, with unit outward normal $\mathbf{n}(\mathbf{x})$, where $\mathbf{x}$ is the coordinate with respect to a Cartesian reference system. Henceforth, given a generic function $f(\mathbf{x},t)$, we use $\overline{f}(t)$ to indicate the volume average of $f(\mathbf{x},t)$ over Ω:

$$\overline{f}(t)=\frac{1}{V}\int_{\Omega} f(\mathbf{x},t)\,\mathrm{d}\mathbf{x} \tag{2.1}$$

Let $\mathbf{u}(\mathbf{x},t)$, $\boldsymbol{\varepsilon}(\mathbf{x},t)$ and $\boldsymbol{\sigma}(\mathbf{x},t)$ be, respectively, the displacement, strain and stress fields at the point $\mathbf{x}\in\Omega$, at the time $t\in T$. The problem on the RVE (Problem P), supposed to be subject, on the boundary, to imposed displacements of the "affine" kind, reads

$$\text{Problem } P \quad \begin{cases} \operatorname{div}\boldsymbol{\sigma}=\mathbf{0} \quad \text{in } \Omega\times T \\ \boldsymbol{\varepsilon}=\operatorname{sym}\nabla\mathbf{u} \quad \text{in } \Omega\times T \\ \mathbf{u}=\overline{\boldsymbol{\varepsilon}}\,\mathbf{x} \quad \text{on } \Gamma\times T \\ \boldsymbol{\sigma}=\mathbf{L}\,\boldsymbol{\varepsilon} \quad \text{in } \Omega\times T \end{cases} \tag{2.2}$$

where $\mathbf{L}$ is the constitutive law operator, supposed to be linear and invertible, div $\boldsymbol{\sigma}$ denotes the divergence of the stress field, while the symbol sym $\nabla\mathbf{u}$ indicates the symmetric part of the gradient of the displacement vector $\mathbf{u}(\mathbf{x},t)$.

The related homogenized constitutive law is

$$\overline{\boldsymbol{\sigma}}(t)=\mathbf{L}^h\,\overline{\boldsymbol{\varepsilon}}(t) \tag{2.3}$$

with $\mathbf{L}^h$ the homogenized counterpart of $\mathbf{L}$. We suppose that the overall operator $\mathbf{L}^h$ satisfies the same properties of the operator $\mathbf{L}$.

Throughout this paper, if not otherwise specified, we will use the following non-degenerate "classical" bilinear form:

$$\langle\boldsymbol{\sigma},\ \boldsymbol{\varepsilon}\rangle=\int_T\left[\frac{1}{V}\int_{\Omega}\boldsymbol{\sigma}(\mathbf{x},t)\,\boldsymbol{\varepsilon}(\mathbf{x},t)\,\mathrm{d}\mathbf{x}\right]\mathrm{d}t \tag{2.4}$$

As it is well-known, the symmetry of problem (2.2) with respect to the bilinear form (2.4) depends only on the symmetry of the constitutive law operator $\mathbf{L}$.[1] In case $\mathbf{L}$ is symmetric with respect to (2.4), the problem admits a "classical" variational formulation; otherwise, some particular procedures may be applied, in order to transform the original problem into a new one ruled by a potential operator (see, for instance, Gurtin (1964), Rafalski (1969a,b), Magri (1974), Leipholz (1979), Telega (1979), Tonti (1984), and Carini and De Donato (2004)).

In this investigation, we apply the method developed by Tonti (1984), which consists in the symmetrization of the original problem through the introduction of a suitable symmetric "integrating" operator, here called $\mathbf{S}$. The advantage of the application of Tonti's method rather than any of the other symmetrization techniques is that, for any given problem, it allows one to derive not just a variational formulation but a minimum principle, if the integrating operator is chosen to be positive definite, a hypothesis we will assume true throughout the paper. In particular, let us suppose that $\mathbf{S}$ is a positive definite operator representing the constitutive law operator of the following problem (Problem P^s):

$$\text{Problem } P^s \quad \begin{cases} \operatorname{div}\boldsymbol{\sigma}^s=\mathbf{0} \quad \text{in } \Omega\times T \\ \boldsymbol{\varepsilon}^s=\operatorname{sym}\nabla\mathbf{u}^s \quad \text{in } \Omega\times T \\ \mathbf{u}^s=\overline{\boldsymbol{\varepsilon}}^s\,\mathbf{x} \quad \text{on } \Gamma\times T \\ \boldsymbol{\sigma}^s=\mathbf{S}\,\boldsymbol{\varepsilon}^s \quad \text{in } \Omega\times T \end{cases} \tag{2.5}$$

where, generally, $\overline{\boldsymbol{\varepsilon}}^s(t)\neq\overline{\boldsymbol{\varepsilon}}(t)$.

The homogenized constitutive law of Problem P^s (2.5) reads

$$\overline{\boldsymbol{\sigma}}^s(t)=\mathbf{S}^h\,\overline{\boldsymbol{\varepsilon}}^s(t) \tag{2.6}$$

where $\mathbf{S}^h$ is the homogenized countepart of $\mathbf{S}$.

The straightforward application of Tonti's method (Tonti, 1984) to Problem P (2.2) yields some disadvantages when bounding the overall properties of the composite. Such drawbacks, due to the direct relation between the variables $\boldsymbol{\varepsilon}(\mathbf{x},t)$ and $\boldsymbol{\sigma}(\mathbf{x},t)$, expressed by (2.2-d), are fully explained in the context of the applications of the method (see Example 1 in Section 4). Therefore, we introduce a new problem (Problem $\tilde{P}$ (2.7)), generalization of the original Problem P (2.2), so that the new unknown functions (stress and strain fields) are not directly related, with the result of avoiding the aforementioned shortcomings. In particular, in Example 1 in Section 4, by virtue of a suitable choice of the new variables, Problem P is recovered and the drawbacks concerning the related bounds are highlighted.

Let us consider then the following new problem:

$$\text{Problem } \tilde{P} \quad \begin{cases} \operatorname{div}\boldsymbol{\tau}=\mathbf{0} \quad \text{in } \Omega\times T \\ \boldsymbol{\varphi}=\operatorname{sym}\nabla\mathbf{v} \quad \text{in } \Omega\times T \\ \mathbf{v}=\overline{\boldsymbol{\varphi}}\,\mathbf{x} \quad \text{on } \Gamma\times T \\ \boldsymbol{\tau}-\boldsymbol{\chi}=\mathbf{L}(\boldsymbol{\varphi}-\boldsymbol{\theta}) \quad \text{in } \Omega\times T \end{cases} \tag{2.7}$$

where the composite is the same of problem P (2.2), but with an imposed stress field $\boldsymbol{\chi}(\mathbf{x},t)$ and an imposed strain field $\boldsymbol{\theta}(\mathbf{x},t)$. Note that, in this case, the stress and strain fields, i.e., $\boldsymbol{\tau}(\mathbf{x},t)$ and $\boldsymbol{\varphi}(\mathbf{x},t)$, are not directly related by the operator $\mathbf{L}$ (due to the arbitrariness of the

[1] Strictly speaking, the symmetry of the whole problem (2.2) should be considered with respect to the bilinear form (2.4) with the addition of a suitable boundary term, such as $\int_T\left[\frac{1}{V}\int_\Gamma\boldsymbol{\sigma}(\mathbf{x},t)\,\mathbf{u}(\mathbf{x},t)\mathbf{n}(\mathbf{x})\,\mathrm{d}\Gamma\right]\mathrm{d}t$. It follows, then, that the divergence operator applied to the stress field proves to be the adjoint operator of the symmetric part of the gradient of the displacement field, by considering also the kinematic boundary conditions (2.2-c). However, if the constitutive law operator $\mathbf{L}$ is not symmetric with respect to (2.4), also the whole problem (2.2) is not endowed with such a property.

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