Loss of ellipticity in elasticity with energy limiters

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Loss of ellipticity in elasticity with energy limiters

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Abstract

Traditional hyperelastic models usually obey requirement of material stability in various forms: Baker-Ericksen inequalities, strong ellipticity, polyconvexity etc. It is reasonable, of course, to require stability for the intact behavior of materials. However, all materials fail and a description of failure should be incorporated in the constitutive law. A simple version of hyperelasticity with failure can be formulated based on the introduction of the limiter in the strain energy density. The limited strain energy bounds maximum achievable stress automatically. Evidently, the elasticity with the energy limiter should exhibit material instability.

This note addresses two practically interesting calculations concerning the onset of material instability via the loss of ellipticity. First, we consider simple shear of natural rubber. We find the direction of failure localization, which is in perfect *qualitative* correspondence with fracture observations in rubber bearings after earthquakes. Interestingly, the direction of failure localization is different from the one predicted by the criterion of maximum tension stress or stretch. Second, we consider equibiaxial tension of a sheet of aneurysm material. We find that the isotropic aneurysm material exhibits infinitely many possible directions of failure localization in equibiaxial tension. The latter means that the random direction of cracks in equibiaxial tension experiment, e.g. membrane inflation, can be an indicator of material isotropy. Accordingly, a preferable direction of the crack alignment can be interpreted as an indicator of the aneurysm anisotropy.

1 Introduction

Materials fail. This physical observation was a bit ignored during the development of the theory of elasticity. Indeed, various requirements (Baker-Ericksen inequalities, strong ellipticity, polyconvexity etc.) are often imposed on the constitutive models to prevent from the appearance of

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