European Journal of Mechanics A/Solids 62 (2017) 1–13

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

A plane strain elasticity model for the acoustical properties of rib-stiffened composite plates

Fengxian Xin [a, b, *], Tian Jian Lu [a, b, **]

[a] State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an, 710049, PR China
[b] MOE Key Laboratory for Multifunctional Materials and Structures, Xi'an Jiaotong University, Xi'an, 710049, PR China

ARTICLE INFO

Article history:
Received 26 July 2016
Received in revised form
1 November 2016
Accepted 4 November 2016
Available online 9 November 2016

Keywords:
Plates
Vibration
Analytical modeling

ABSTRACT

A theoretical elasticity model is established to study the noise reduction performance of a compliant coating layer that is attached to a periodically rib-stiffened plate under time-harmonic mechanical excitation. The general case of the structure immersed in two different fluids is considered so as to accurately simulate the interior and exterior fluid media of hull structures in underwater environment. The theory of plane strain elasticity is employed to model the dilatational and shear motions of the compliant coating layer and the elastic plate, while the scalar Helmholtz equation is adopted to describe the motions of the two fluids. Vibroacoustic coupling is realized by enforcing displacement and stress continuity at adjacent layer interfaces, with the reaction forces of the rib-stiffeners accounted for by introducing them as discretely distributed stresses. The resultant boundary value equations of the system are solved by applying the Fourier transform technique, based upon which the noise reduction due to the compliant coating layer can be favorably calculated. Numerical investigations are implemented to explore the effects of coating thickness, coating material properties, radiation angle, and excitation location on noise reduction. The theoretical results presented in this study provide valuable guidance for experimental research and structural design related to the decoupling effects of coating layers affixed to elastic structures in underwater environment.

1. Introduction

The combined mechanical and acoustic performance of composite structures is of great concern in many engineering applications, since these structures are often subjected to dynamic and acoustic excitations (Ehsan Omidi, 2016; Li et al., 2013; Li and Lyu, 2014; Reza Ansari, 2016; Xin and Lu, 2016; Chen and Huang, 2015). Among these, noise reduction by compliant coating layers attached to underwater structures for enhanced structure sound proofing has received increasing attention (Hladky-Hennion and Decarpigny, 1991; Keltie, 1998; Tao et al., 2010). The coating layer made of a relatively soft material contributes to enlarge the thickness deformation over the elastic structure under fluid loading, causing therefore the decoupling of the motions of the elastic structure and the surrounding fluid (Berry et al., 2001; Foin et al., 2000). The acoustic design of such coated structures calls for accurate theoretical modeling of the noise reduction response of the compliant coating layer. The purpose of this paper is therefore to develop a theoretical elasticity model for noise reduction achieved by attaching a compliant coating layer to a periodically rib-stiffened plate immersed in fluids.

A great deal of work has been devoted to studying the mechanical and acoustical performance of composite structures. For instance, a layerwise/solid-element approach was proposed to analyze the linear static and free vibration of composite sandwich plates (Hasheminejad and Miri, 2007; Li et al., 2013; Pandey and Pradyumna, 2015; Sadeghpour et al., 2016; Sahoo et al., 2016). The soundproofing effect of particle composites was experimentally studied, and concluded that the transmission loss of the composites was increased nonlinearly with increasing filler volume fraction (Liang and Jiang, 2012). Also, the mechanical, thermal and acoustical performances of composites were experimentally investigated from the viewpoint of manufacturing conditions

* Corresponding author. State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an, 710049, PR China.
** Corresponding author. State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an, 710049, PR China
E-mail addresses: fengxian.xin@gmail.com (F. Xin), tjlu@mail.xjtu.edu.cn (T.J. Lu).

http://dx.doi.org/10.1016/j.euromechsol.2016.11.001
0997-7538/

(David and Francesco, 2015; Hao et al., 2013).

Actually, the Kirchhoff thin plate theory was commonly adopted to model the vibration and acoustic responses of single- or multi-layer panel configurations (Keltie, 1993; Legault and Atalla, 2010; Lin and Garrelick, 1977; Mace, 1980; Narayanan and Shanbhag, 1981; Xin and Lu, 2010a). To account for the influence of shearing deformation and rotary inertia, the Mindlin plate theory was developed for modeling thick plates (Gholami et al., 2016; Mindlin, 1951; Park and Mongeau, 2008). To more accurately describe three-dimensional (3D) displacements and stresses, the elasticity theory of solids should be employed to handle the dilatational and shear wave behaviors of solid layers (Ko, 1997; Maidanik and Tucker, 1974; Qu and Meng, 2014). This is of vital importance for obtaining exact predictions at higher wavenumbers and frequencies, as plate theories failed to do so.

Adopting the elasticity theory, Jackins and Gaunaurd formulated a theoretical model for sound reflection from multilayered elastic flat plates (Jackins and Gaunaurd, 1986), while Chonan and Kugo conducted an acoustic design for three-layered plates with high sound interception in terms of various combinations of the facing and the core materials (Chonan and Kugo, 1991). Ko established a theoretical model to evaluate the effectiveness of an air-voided elastomer layer perfectly bonded to an infinite plate in reducing the structure-borne noise (Ko, 1997). Often, to deal with plate structures covered with coating layers, the elasticity theory was also combined with the thin plate theory, yielding sufficiently accurate results. For instance, focusing on the signal velocity response of elastically coated plates, Keltie developed an analytical model for a compliant elastic coating attached to a submerged thin plate (Keltie, 1998). Modeling the decoupling layer with 3D elasticity theory, Berry et al. presented a theoretical analysis for the vibroacoustic response of a finite simply-supported rectangular plate covered by a decoupling layer and immersed in heavy fluid.

Although the sound radiation and transmission issues of elastically coated plate structures have been extensively studied, the noise reduction realized by attaching a compliant coating layer to a plate structure (periodically rib-stiffened plate in particular) is rarely studied. Since the relationship between noise reduction and sound transmission loss has not been clearly explained (Tao et al., 2010), an individual investigation for the noise reduction behavior of the compliant coating layer is of paramount importance to understand the soundproofing effectiveness of the coating. This study aims to squarely address this issue. By utilizing the Fourier transform technique, a 3D elasticity model is proposed to explore the noise reduction of a compliant coating layer affixed to a rib-stiffened plate. The model is capable of dealing with the case that the structure is immersed in two different fluids on its two sides, which thus can favorably simulate the real environment of underwater structures. For illustration, numerical investigations are conducted to quantify the effects of key system parameters (e.g., coating thickness, radiation angle, coating material properties and excitation location) on the noise reduction of the compliant coating layer.

2. Theoretical formulation

2.1. Structure dynamics of plate and coating

With reference to Fig. 1, consider a periodically rib-stiffened elastic plate of infinite extent in the *xz* plane covered with a uniform coating layer of a compliant, elastic solid on the upper surface. These rib-stiffeners are parallelly connected with the elastic plate along the z-direction, separated by a constant distance of L in the x-direction. The entire structure is immersed in two different fluids, i.e., fluid 1 on the lower half-space and fluid 2 on the upper half-space. The thicknesses of the compliant coating layer and the elastic plate are represent by the symbols h_t and h_b, while the height and thickness of the rib-stiffeners are denoted by h_r and t, respectively. A line time-harmonic force $f(x,t)$ along the z-direction is assumed to exert on the bottom of the elastic plate, causing the vibration of the structure and sound radiation in both the lower and upper half-spaces.

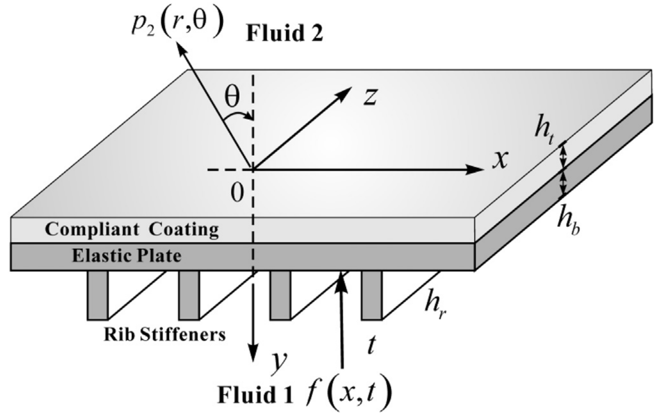

Fig. 1. Schematic illustration for noise reduction of a rib-stiffened plate covered with a compliant coating layer.

Both made of linear, isotropic and homogeneous solid materials, the motions of the compliant coating layer and the elastic plate should obey the Navier-Cauchy equation in the category of elasticity theory, as:

$$\mu_a \nabla^2 \mathrm{u}^{(a)} + (\lambda_a + \mu_a)\nabla\left(\nabla\cdot\mathrm{u}^{(a)}\right) = \rho_a \frac{\partial^2 \mathrm{u}^{(a)}}{\partial t^2} \tag{1}$$

where $\mathrm{u}(x,y,t)$ is the displacement vector, λ_a and μ_a are the Lamé constants, ρ_a denotes alternatively the density for the compliant coating layer and the elastic plate when its subscript $a = t$ and $a = b$. The same meaning of the subscript holds throughout the study. The stresses and strains in the elastic solid layers comply with the following constitutive relationship:

$$\sigma_{ij}^{(a)} = \lambda_a\left(\nabla\cdot\mathrm{u}^{(a)}\right)\delta_{ij} + \mu_a\left(u_{i,j}^{(a)} + u_{j,i}^{(a)}\right) \tag{2}$$

To describe the dilatational motion and shear motion of a solid, the elasticity theory dictates that the displacement of the solid can be divided into the irrotational part and the rotational part (corresponding separately to dilatational motion and shear motion), as:

$$\mathrm{u}^{(a)} = \nabla\phi^{(a)} + \nabla\times\psi^{(a)} \tag{3}$$

where $\phi^{(a)}$ is the dilatational scalar potential and ψ is the shear vector potential. Given the two-dimensional (2D) plane strain nature of the system, the shear vector potential ψ actually has only one component in the z-direction, i.e., taking the form of $(0, 0, \psi^{(a)})$. The two potentials are separately governed by the Helmholtz equation:

$$\nabla^2\phi^{(a)} = \frac{1}{\left[c_d^{(a)}\right]^2}\frac{\partial^2\phi^{(a)}}{\partial t^2}, \quad \nabla^2\psi^{(a)} = \frac{1}{\left[c_s^{(a)}\right]^2}\frac{\partial^2\psi^{(a)}}{\partial t^2} \tag{4}$$

in which the dilatational wave speed and the shear wave speed are determined by the Lamé constants and the density of the solid material, as: