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Core-shell spheres under diametrical compression: An analytical solution

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ABSTRACT

The analytical solution for the stress distributions within core-shell spheres or layered spheres under diametrical compression is obtained. The solution reduces analytically to the classical solution for solid spheres by Hiramatsu and Oka (1966) and the analytical solution for hollow spheres by Wei et al. (2015) in the two limiting special cases. The numerical results of the present solution show that a drastic increase of tensile stress is usually observed for a core-shell sphere with a stiff shell or a shell with the same stiffness but a small Poisson's ratio, while a drastic decrease of tensile stress is usually observed for a core-shell sphere with a soft shell or a shell with the same stiffness but a large Poisson's ratio. The maximum tensile stress may be either induced at the interface or near the loading areas, which is more likely induced at the interface of a core-shell sphere with a thin and stiff shell, otherwise it is more likely induced near the loading areas of a core-shell sphere. Moreover, the maximum tensile stress is affected by the size of loading areas, the ratio of the Young's moduli of the core and the outer shell, Poisson's ratio and the thickness of the shell. Since more and more composite materials made up of core-shell spheres are used for some advanced devices to achieve multi-functions or some intelligent abilities, the present solution can be used as a benchmark or a basic solution for analyzing the failure mechanism of composite materials made up of core-shell spheres or layered spheres.

1. Introduction

It is in recent years that core-shell spheres or layered spheres have attracted more and more interests because of their promising applications for nanoscale chemical reactors, catalysts, drug-delivery carriers, and photonic building blocks. Various kinds of new technologies have been induced to synthesize core-shell spheres or layered spheres with a wide range of diameter and wall thickness, which have been eventually used in the fields of medical science, chemistry, biotechnology, electronic materials and civil engineering. For example, NiAl-layered microspheres are made for electrochemical detection (Shen et al., 2016). SI@C/RGO core-shell sphere exhibits enhanced electrochemical performance with high reversible specific capacity (Lin et al., 2015a,b). It was found that Li-rich layered microspheres show high capacity and superior rate-capability and prepared by a kind of slurry spray-drying process (Fan et al., 2015; Hou et al., 2015). Core-shell spheres are synthesized as chiral metamaterial of different parameters and the electromagnetic scattering properties are analyzed (Salam et al., 2015). The layered CoFe spheres are used for the electrocatalytic oxygen reduction reaction (Zhang et al., 2015). Ni–Fe double hydroxide/MnO_2 core-shell sphere are synthesized and used as an efficient noble metal-free electrocatalyst (Jia et al., 2015). Magnetic core-shell microspheres are very useful for removal of dihydroxybenzoic acid from aqueous solutions (Tang et al., 2015). Layered microspheres are made for superior electric capacitors (Kim et al., 2015). ZnAl layered microspheres compounded with core-shell structure show considerable enhanced adsorption capability (Lin et al., 2015a,b), and tripartite Core-shell spheres are used as a kind of enhanced efficiency dye sensitized solar cells and other materials (Cheng et al., 2013; Sorribas et al., 2012). Size, shape effects, and elastic properties of core-shell nanoparticles were studied (Rinaldi et al., 2010; An and Liu, 2013).

Evidence shows that composite materials made up of above core-shell spheres or layered spheres often suffer mechanical and volume changes during the process of achieving above various kinds of advanced functions, and local failure and micro-cracks

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have been observed (Zhao et al., 2011; Golmon et al., 2010). In order to understand the failure mechanism of the composite materials made up of core-shell spheres, it is vital to know the mechanical properties of the individual core-shell sphere or layered sphere under diametrical compression, which is also the simplest but a typical case representing the interactions between the neighbor core-shell spheres or layered spheres.

The diametrical compression of solid and hollow spheres is one of the most fundamental problems in applied mechanics, and has been studied extensively because of its scientific and engineering interests. It has been used for testing the deformability, the hardness, the crushing strength, and other mechanical properties of brittle materials. The related studies are for solid spheres (Hiramatsu and Oka, 1966; Shipway and Hutchings, 1993; Chau and Wei, 1999; Chau et al., 2000; Wei, 2009) as well as for hollow spheres (Taber, 1983; Gregory et al., 1998; Gupta et al., 1999; Wan and Liu, 2001; Dong et al., 2008; Shorter et al., 2010). Particularly for analytical solutions for solid and hollow spheres, Gregory et al. (1998) obtained an asymptotic solution for a thick hollow sphere compressed by equal and opposite concentrated axial loads. Hiramatsu and Oka (1966) derived an analytical solution for isotropic solid spheres under diametrical point loads and provided the classical basis for the point load strength test. Chau and Wei (1999) and Wei (2009) extended the classical solution for solid spheres with spherically and cubic anisotropy, respectively, and the solutions reduce analytically to the classical solution by Hiramatsu and Oka (1966) and agree well with experimental results by Frocht and Guernsey (1953) in isotropic limiting case. Wei et al. (2015) recently provided a relatively comprehensive review on experimental and theoretical studies on hollow spheres and obtained an analytical solution for hollow spheres under the diametrical point loads.

All of the previous studies are either for solid spheres or hollow spheres under diametrical compression, and (to best of our knowledge) there is no analytical solution for core-shell spheres or layered spheres under diametrical compression. Therefore, in this paper, the general solution form for the stress components and the displacement components recently derived by Wei et al. (2015) are used as basic solution form for stress distributions for the internal cores and the outer shells of the core-shell spheres, and the boundary conditions of the pair of diametrical forces acting on two small areas of the outer surface of the core-shell sphere and the bonding conditions at the interface are used to determine the unknown coefficients in the general expressions. Numerical results for the stress distributions within the core-shell spheres will be obtained, and the stress concentrations induced by the interface between the inner core and the out shell will be analyzed in detail. The present solution can be considered as an extension of the classical solution by Hiramatsu and Oka (1966) for solid spheres under the point loads, which provides theoretical basis for the Point Load Strength Test (PLST) and has been recognized as an extremely useful index test in engineering for rock classification and strength estimation. The present solution, however, can be used as a basic solution as well as a benchmark for numerical solutions for studying the failure mechanism of bulk composite materials made up of core-shell spheres or layered spheres.

2. Governing equations and the boundary conditions for the layered solid spheres

The core-shell spheres or layered solid sphere considered is made up of an elastic core and an elastic shell, and the origin of the spherical coordinate system (r,θ,φ) is set up at the center of the core-shell spheres as shown in Fig. 1. The relations between the components of stress $\boldsymbol{\sigma}$ and strain $\boldsymbol{\varepsilon}$ are expressed by the Hooke's law for isotropic core-shell spheres as

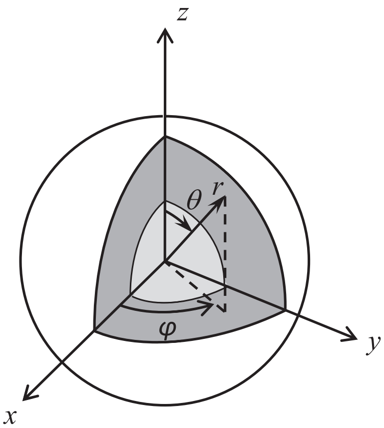

Fig. 1. A sketch for a core-shell sphere with a cut section showing the spherical coordinate system (r,θ,φ).

$$\sigma_{rr}^i=\lambda^i\left(\varepsilon_{rr}^i+\varepsilon_{\theta\theta}^i+\varepsilon_{\varphi\varphi}^i\right)+2G^i\varepsilon_{rr}^i,\quad \sigma_{\theta\theta}^i=\lambda^i\left(\varepsilon_{rr}^i+\varepsilon_{\theta\theta}^i+\varepsilon_{\varphi\varphi}^i\right)+2G^i\varepsilon_{\theta\theta}^i,$$
$$\sigma_{\varphi\varphi}^i=\lambda^i\left(\varepsilon_{rr}^i+\varepsilon_{\theta\theta}^i+\varepsilon_{\varphi\varphi}^i\right)+2G^i\varepsilon_{\varphi\varphi}^i,\quad \tau_{\theta\varphi}=G^i\gamma_{\theta\varphi}^i,\quad \tau_{r\theta}^i=G^i\gamma_{r\theta}^i,$$
$$\tau_{r\varphi}^i=G^i\gamma_{r\varphi}^i \tag{1}$$

where $i=I$ and II represent for the inner core and the outer shell, respectively. λ^i and G^i are the Lame's constants of the core-shell sphere, and they are related to the Young's modulus E^i and Poisson's ratio ν^i by

$$\lambda^i=\frac{E^i\nu^i}{(1+\nu^i)(1-2\nu^i)},\qquad G^i=\frac{E^i}{2(1+\nu^i)} \tag{2}$$

Neglecting the body forces, the equilibrium equations for the core and the shell of the core-shell sphere can be written respectively as (Timoshenko and Goodier, 1982)

$$\frac{\partial\sigma_{rr}^i}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta r}^i}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\tau_{r\varphi}^i}{\partial\varphi}+\frac{1}{r}\left(2\sigma_{rr}^i-\sigma_{\theta\theta}^i-\sigma_{\varphi\varphi}^i+\tau_{r\theta}^i\cot\theta\right)=0$$
$$\frac{\partial\tau_{r\theta}^i}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^i}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\tau_{\theta\varphi}^i}{\partial\varphi}+\frac{1}{r}\left[\left(\sigma_{\theta\theta}^i-\sigma_{\varphi\varphi}^i\right)\cot\theta+3\tau_{r\theta}^i\right]=0$$
$$\frac{\partial\tau_{r\varphi}^i}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta\varphi}^i}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\sigma_{\varphi\varphi}^i}{\partial\varphi}+\frac{1}{r}\left(3\tau_{r\varphi}^i+2\tau_{\theta\varphi}^i\cot\theta\right)=0 \tag{3}$$

The relation between the strain components and displacement components for the core and the shell of the core-shell sphere can be obtained respectively as

$$\varepsilon_{rr}^i=\frac{\partial u_r^i}{\partial r},\quad \varepsilon_{\theta\theta}^i=\frac{1}{r}\frac{\partial u_\theta^i}{\partial\theta}+\frac{u_r^i}{r},\quad \varepsilon_{\varphi\varphi}^i=\frac{1}{r\sin\theta}\frac{\partial u_\varphi^i}{\partial\varphi}+\frac{u_r^i}{r}+\frac{u_\theta^i}{r}\cot\theta,$$
$$\gamma_{r\varphi}^i=\frac{1}{r\sin\theta}\frac{\partial u_r^i}{\partial\varphi}-\frac{u_\varphi^i}{r}+\frac{\partial u_\varphi^i}{\partial r},\quad \gamma_{r\theta}^i=\frac{1}{r}\frac{\partial u_r^i}{\partial\theta}-\frac{u_\theta^i}{r}+\frac{\partial u_\theta^i}{\partial r},$$
$$\gamma_{\theta\varphi}^i=\frac{1}{r}\frac{\partial u_\varphi^i}{\partial\theta}-\frac{u_\varphi^i}{r}\cot\theta+\frac{1}{r\sin\theta}\frac{\partial u_\theta^i}{\partial\varphi} \tag{4}$$

The boundary conditions for an elastic core-shell sphere under the compression of a pair of diametrical resultant forces F can be