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Numerical analysis of plane stress free vibration in severely distorted mesh by Generalized Finite Element Method

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A B S T R A C T

This paper presents a C^0 quadrilateral enriched element by the Generalized Finite Element Method, having trigonometric and exponential functions as enrichment functions, applied in free vibration analysis with distorted mesh. The stiffness and mass matrices are obtained by subintervals numerical integration. The efficiency of the enriched C^0 element is observed by solving several plane stress free vibrations problems. The analyses include uniform and non-uniform mesh models, and severely distorted meshes. Furthermore, the sensitivity of the enriched C^0 element is also analyzed. The results of the analyses are compared with other numerical formulations and show that the enriched C^0 element proposed in this paper has good performance.

1. Introduction

The dynamic characterization of structures or a continuous medium depends on the correct determination of their frequencies and natural modes of vibration. The numerical evaluation of these characteristics involves the solution of an eigen problem, where the eigen pairs are the natural frequencies and the vibration modes. The current literature shows many difficulties in accurately determining the higher orders frequencies, which can compromise the quality of dynamic response - see, for example, Cottrell et al. (2006), Torii et al. (2015); Arndt et al. (2016).

To overcome these difficulties in finite element analysis, the usual solution is to refine the mesh (*h*-refinement) or increasing the polynomial order of the elements (*p*-refinement). However, even with the increase in the total number of degrees of freedom by these refinements, yet there is significant loss of accuracy at higher frequencies. Hughes et al. (2005) show that the conventional finite element solution has good accuracy in the acoustic branch of the frequencies spectrum, but there is a degradation in the optical branch. The limit of acoustic and optical branches for linear elements, for example, is near of 50% of the frequency spectrum. This means that, in a dynamic analysis, only the lower 50% modes, those of acoustic branch, have the necessary precision to well represent the dynamic behavior of the structure or continuous medium.

Enriched methods based on the Partition of Unity Method, as the generalized (GFEM) and extended (XFEM) finite element methods, show better performance than conventional finite element method in the determination of the eigenvalues and eigenvectors. The limit between the acoustic and optical branches is higher than conventional finite elements (Hsu, 2016). Consequently, a greater number of accuracy modes is calculated in the analysis and the error is minimized (Melenk and Babuska (1996); Duarte and Oden (1996), Belytschko et al. (2009); Arndt et al. (2010)). Equivalence between GFEM and XFEM methods was observed by Belytschko et al. (2009) and, therefore, from now on, this work will use the name GFEM in the place of XFEM.

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Another advantageous aspect of the enriched methods is its hierarchical nature. New enrichment functions can be added to construct enriched mathematical space without mesh reconstruction. Thus, there is no need of rebuilding mass and stiffness matrices of previous level of enrichment for each addition of new enrichment level, unlike the conventional FEM. Despite these advantages, enriched methods present certain difficulties that require special attention. The numerical integrations often require a greater number of integration points. The numerical stability of equations systems also depends on the number of significant digits used. Moreover, the elements are more sensitive to mesh distortions (Arndt et al., 2010; Torii et al., 2015). This study investigates some of these aspects.

To better understand the conceptual aspects of the GFEM for dynamic analysis, we now present briefly a literature review on enriched finite element formulations, such as the hierarchical formulations and the formulations based on the partition of unity (PU) method.

The hierarchical enrichment formulation was presented by Zienkiewicz et al. in 1983 as an enriched method of conventional finite elements, which later came to be known as the hierarchical finite elements method (HFEM). The idea of this method is based on adding hierarchical functions in the shape functions without the necessity to change the previous ones.

These characteristics of the HFEM favor the development of adaptive meshes and its applications to solve boundary value problems. The automatic process in adaptive meshes is controlled by error estimation (Deuflhard et al., 1989). In the context of elastodynamic analysis, the hierarchical method was applied, among the others, by Belytschko et al. (1995), for crack propagation problem, and by Cho and Youn (1995) and Cramer et al. (1999).

For hierarchical enrichment, several mathematic functions have been considered and applied in many situations. *B-splines* functions were adopted by Leung and Au (1990) in the place of polynomial functions for beam and plate elements. From the applications of this method, the computational processing time was reduced, while it maintains the accuracy of the results. Trigonometric functions were also adopted as hierarchical functions (Ganesan and Engels, 1992), as well as Bardell's functions (Han and Pety, 1996a; b) for analysis of free vibration of plates. Fourier series were also adopted for the enriched beam and plate element by presenting a well-conditioned matrix as compared to other functions (Leung and Chan, 1998). Moreover, Ribeiro and Petyt (1999) presented the hierarchical formulation for geometrically nonlinear dynamic analysis. For hierarchical formulation of the enriched quadrilateral C^0 element, several functions have been proposed, such as the Legendre (Yu et al., 2010), Lobatto, Kernel (Solin et al., 2004), and Fourier series (Leung et al., 2004) functions.

Among the hierarchical formulations, the Generalized Finite Elements has highlighting out. The proposal of this enriched finite element method was given by Babuska et al. (1995), when it was presented as the Partition of Unity Finite Element Method (PUFEM). Independently, similar concepts were presented by Duarte and Oden (1996) and by Oden et al. (1998), as a new cloud based *hp*-finite element method. Its employment under the current name of the GFEM arises, however, for the first time in Melenk and Babuska (1996).

The GFEM is based on the Partition of Unity Method with conventional FE shape functions as partition of unity (PU). [(Melenk and Babuska, 1996), (Duarte and Oden, 1996) (Strouboulis et al., 2000),]. Specific characteristics of the problem can be incorporated in the local approximation functions, which are used to enrich the space of solution. The enriched FE shape functions are canceled at conventional finite element nodal points and the main features of conventional finite element method are preserved. The accuracy of the numerical integration for the stiffness matrix is controlled according to the type of function introduced and system of equations linearly independents can be solved by using known numerical methods (Strouboulis et al., 2000).

The generalized finite element approach has been applied in many engineering problems, including, for example, dynamic crack propagation. In this case, elements through which the crack propagates are enriched with special functions, which may represent the stress intensity effect. The enrichment functions may be polynomials or other kind of functions already known as, for example, the equation for calculating the stress intensity factors. The stress field around the crack tip is good represented by GFEM [(Duarte et al., 2001), (Bui and Zhang, 2012), (Yu et al., 2016), (Sharma et al., 2013)]. Alternatively, the global local approach has also been used, which enriches specific regions of the mesh only after a first solution with coarse conventional elements meshes (Gupta et al., 2012; Duarte and Kim, 2008). Other applications with GFEM are the adaptive mesh for analysis of damage mechanics (Barros et al., 2004), elastohydrodynamic analysis and free vibrations, in beams or plates, or states of plane stress (De Bel et al., 2005; Arndt et al., 2010). A *posteriori* error estimation was also developed by Strouboulis et al. (2006). The GFEM has also been applied to solve the Helmholtz equation, which describes physical problems such as acoustic and electromagnetic problems, among others. In some applications, exponential and trigonometric functions were adopted as enrichment functions in problems that involve the propagation of two-dimensional waves. The employment of trigonometric and exponential function as enrichment function in dynamic analysis is not new [(Arndt et al., 2010), (Leung and Chan, 1998), (Leung et al., 2004) (Torii et al., 2015),]. Due to the dynamic response presents curve profile similar to trigonometric and exponential functions. Furthermore, this employment is a novelty in the enriched C^0 element formulated by GFEM. The choice of enrichment functions should respect criteria established by the GFEM. These criteria are presented in the literature (Strouboulis et al., 2008).

However, some researchers (Babuska and Banerjee, 2012; Zhang et al., 2014) have reported that the GFEM may have high values for condition number of stiffness matrix. To overcome this problem, the authors presented a small adjustment in the GFEM to make it a more efficient method. This modification received the name of *Stable Generalized Finite Element* (SGFEM). The new modification, whose results were satisfactory for structural analysis, preserves the features of the GFEM, and introduces modifications and other selection criteria for the enrichment function.

It is worth mentioning that the enriched formulation of the quadrilateral domain was also developed by the meshless method [(Bui et al., 2011) (Bui et al., 2013),], isogeometric approach (Thai et al., 2014), adaptive modified element formulation [(Nguyen et al., 2013), (Nguyen et al., 2015), (Nguyen et al., 2016)], spectral finite element method [(Song et al., 2016), (Li and Soares, 2015), (Park and Lee, 2015), (Hedayatrasa et al., 2014), (Shirmohammadi et al., 2015), (Wang and Unal, 2013), (Joglekar and Mitra, 2016)], and other methods derived from the Partition of Unity Method (PUM), whereas a local approximation function can be of various categories, including the least square point interpolation method (LSPIM) (Rajendran and Zhang, 2007; Zhang et al., 2014). Additionally, studies also show the versatility of adopting a local radial polynomial approximation function (Xu and Rajendran, 2011). In the PUM context, there are other formulations that adopt different mathematical function, such as spline functions (Chen et al., 2010), or the consecutive interpolation method, in order to avoid recomputing the quadrilateral element shape function [(Bui et al., 2014), (Bui et al., 2016) (Kang et al., 2015),].

A relevant aspect for good performance of GFEM is the