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On sequentially coupled thermo-elastic stochastic finite element analysis of the steel skeletal towers exposed to fire

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A B S T R A C T

This work demonstrates an application of the stochastic perturbation technique and the corresponding Stochastic Finite Element Method (SFEM) in numerical analysis of the temperatures, stresses and deformations for the spatial steel tower structure exposed to a fire. This approach is based on the 10th order Taylor expansion of all input random parameters, the resulting state functions and, independently, on the Least Squares Method (LSM) determination of the structural responses in addition to the input random parameters. An initial temperature equivalent to the fire exposure of the tower structure is adopted as the input Gaussian random variable and applied at the bottom structural members, where material parameters of the steel as Young modulus, heat conductivity and capacity as well as thermal elongation are considered all as highly temperature-dependent. We explore various algebraic forms of the response functions as polynomial bases and, additionally, power, exponential, hyperbolic to make our results totally independent of this choice. This study is an example of a hybrid usage of the Finite Element Method (FEM) system ABAQUS and the computer algebra system MAPLE in stochastic transient sequentially coupled thermo-elastic analysis, where up to the fourth order probabilistic characteristics of the temperatures, displacements and stresses may be directly used in fire reliability analysis of the civil engineering structures with the temperature both independent and dependent material characteristics.

1. Introduction

It is well known that the exposition of various structures to a fire is decisive for their durability and may result in some catastrophic failures, especially in case of steel, aluminum and their hybrid structures. It is clear that the total time from a fire ignition up to this failure strongly depends on the basic structural materials as well as on the protection covers and systems (Dwaikat et al., 2011; Tuśnio, 2010). This sensitivity leading to stability loss, yielding or other forms of global or local failure is still a subject of many experimental, theoretical and numerical validations, even in a pure deterministic context. We are particularly interested in the fire resistance and protection of the steel towers, because they appeared recently to be extremely popular in civil engineering, they are and will be still modified in the nearest future towards the new technological demands and they dominate spatial towers area.

Considering experimental works and overall experience with fire (Dwaikat et al., 2011) it seems to be quite natural to assume that the maximum temperature range during such an exposure is treated as the basic input random variable in transient analysis of the thermal stresses. It needs to be highlighted that the structural steel has so large heat conduction coefficient that for smaller scale structures it is even not mandatory to solve numerically the relevant coupled thermo-elastic problems, where transient heat transfer equation is accompanied with the additional static equilibrium, because the temperature distribution is almost uniformly increased throughout structural elements. Otherwise, we need to consider some scenario or a variety of different scenarios of the fire having practical importance. In case of high skeletal towers the most dangerous situation of that nature is when the fire starts at the terrain level and then propagates on lower sections of the tower only; it is equivalent to the frequent situation of the fire accidents on rural areas. As it is known from the *Eurocode*, some of material parameters may accidentally exhibit a singular character for the specific temperatures (like heat capacity), so that probabilistic and even deterministic FEM analyses are clearly non-trivial in such a case. Relatively large random dispersion of the fire temperature is

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completely new problem in stochastic computational mechanics as material properties or the external load usually have extreme coefficients of variation equal to 0.15 and the one adjacent to the fire temperature may be close to or even larger than 1.0, which makes some lower order methods quite inapplicable here. One can see totally different approach to the fire conditions in (Maślak, 2008) based on Bayesian probability analysis. It should be also mentioned that there is remarkable research gap in literature because there is no papers about stochastic nature of the fire and stochastic analysis including up 4th probabilistic moments. The readers interested in concurrent stochastic computational techniques related to material and geometrical uncertainty may find some interesting information in (Stefanou and Papadrakakis, 2004), for instance. Let us note that our preference concerning computational technique follows also the fact that stochastic perturbation technique incorporates determination of the sensitivity gradients necessary in numerical optimization analyses.

The stochastic method proposed here is based on the 10th order Taylor series expansion with random coefficients about the mean values of the input parameter and similar expansions for all the resulting state functions (Kamiński, 2013). It is related to the Weighted Least Squares Method (WLSM), where the analytical functions relating extreme temperatures, displacements and/or stresses with the input random parameter are numerically recovered thanks to the set of FEM experiments carried out in the system ABAQUS; the randomized Gaussian temperature is gradually changed about its mean value taken from the design fire curve. We use sequentially coupled temperature-displacement model of the structure in several deterministic tests, where transient heat transfer is solved first; then we solve for the linear elasticity structural problem with thermally dependent material characteristics. It should be mentioned that mechanical boundary conditions are entirely deterministic in this study, because the fire phenomenon does not affect the character of the supports, which are completely stiff during the fire. However, a thermal transient process is solved with random boundary condition in the context of the given surface temperature for lower structural elements. Several trial values of the structure surface temperature subjected to a fire and the corresponding extreme values of the state parameters (temperature, displacement and stress) lead via the LSM to four totally different response functions, which after symbolic partial differentiation with respect to random temperature are embedded into the final equations for the additional probabilistic moments and coefficients. We verify the power, exponential, hyperbolic and polynomial responses to make our stochastic coupled problem entirely free from modeling error resulting from the specific algebraic form of this function. The additional statistical evaluation of various response functions has been done and it includes a verification of the correlation coefficient of the approximating curve to the set of given FEM experiments as well as numerical determination of the variance and mean square error of the fitting procedure. The original character of this problem and its numerical solution is in application of the 10th order stochastic perturbation technique (Kamiński and Świta, 2011) to the coupled problem of thermal stresses with randomized boundary conditions according to the Gaussian distribution as well as in application of the response functions of various algebraic structures. It should be mentioned that the usage of the generalized stochastic perturbation technique has been also compared with the Monte-Carlo simulation statistical estimators as well as with the probabilistic characteristics calculated via the semi-analytical technique. It consists in determination of the resulting moments from integral definition by application of the response functions determined with the use of the WLSM strategy.

2. Probabilistic background

Let us introduce random variable b and its probability density function (PDF) as $p_b(x)$. Then, the first two probabilistic moments of this variable are defined as (Kamiński, 2013; Vanmarcke, 1983)

$$E[b] = \int_{-\infty}^{+\infty} b p_b(x) dx, \tag{1}$$

and

$$Var(b) = \int_{-\infty}^{+\infty} (b - E[b])^2 p_b(x) dx. \tag{2}$$

Higher probabilistic moments and related coefficients may be defined according to classical definitions of the probability theory. The basic idea of the stochastic perturbation approach employed here is to expand all input variables and all the resulting state functions of the given thermoelasticity problem via Taylor series about their spatial expectations using the perturbation parameter ε (traditionally taken as equal 1 (Kleiber and Hien, 1992)). The following expression is used (Kamiński, 2013) to provide such an expansion of $T = T(b)$:

$$T(b) = T^0\left(b^0\right) + \sum_{n=1}^{\infty} \frac{1}{n!} \varepsilon^n \frac{\partial^n T(b)}{\partial b^n}\bigg|_{b=b^0} (\Delta b)^n, \tag{3}$$

where

$$\varepsilon \Delta b = \varepsilon\left(b - b^0\right) \tag{4}$$

is the first variation of random parameter b about its average value. Let us analyze further expected values of any state random process $T(t,b)$ defined analogously to the formula (3) by its Taylor series expansion for the given time t

$$E[T(t,b)] = \int_{-\infty}^{+\infty} T(t,b) p_b(x) dx = \int_{-\infty}^{+\infty} \left(T^0\left(t,b^0\right) + \sum_{n=1}^{\infty} \frac{1}{n!} \varepsilon^n \frac{\partial^n T(t,b)}{\partial b^n}\bigg|_{b=b^0} \Delta b^n \right) p_b(x) dx \tag{5}$$

Let us remind that this power expansion is valid only if the state function is analytic in ε and the series converge; therefore, any convergence criteria should include the magnitude of the perturbation parameter (Hien and Kleiber, 1997; Kamiński, 2013). It yields for the input random variable with the Gaussian probability density function

$$E[T(t,b)] = T^0\left(t,b^0\right) + \frac{1}{2} \frac{\partial^2 (T(t,b))}{\partial b^2}\bigg|_{b=b^0} \mu_2(b) + \ldots + \frac{1}{10!} \frac{\partial^{10} (T(t,b))}{\partial b^{10}}\bigg|_{b=b^0} \mu_{10}(b) \tag{6}$$

where $\mu_n(b)$ denotes nth order central probabilistic moment of b (Kamiński, 2013) and all the derivatives are calculated at the average value of the parameter b. We apply further the linearized version of the stochastic perturbation technique, so that the mean value of the Gaussian input variable is assumed to be equal to its