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Study on the band gaps of phononic crystal pipes with alternating materials in the radial and axial directions

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HIGHLIGHTS

- Several new complex cells are designed.
- The new complex cells bring different band structure characteristics. The suggestion given by the Bragg-type band gap mechanism are corrected and improved.
- The new Bragg-type phononic crystal pipes with a same lattice constant obtain a lower frequency band gap or widen the low-frequency band gap.

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ABSTRACT

Low-frequency vibration control is a key technology in the vibration control of pipe systems. The existing related research on band gap mechanisms suggests that gigantic structures are required for the Bragg-type band gaps to control low-frequency vibration. Thus, the application of Bragg-type phononic crystal pipe is greatly restricted. However, the suggestion given by the existing Bragg-type band gap mechanisms is based on simple cell structure and material composition. The cell structures of Bragg-type phononic crystal pipes are also simple and the potential action of the cell has not been fully excavated at present. In this paper, several new cells with relatively complex structure are designed to study the influence on the Bragg-type band gap characteristics. Band structures of the Bragg-type phononic crystal pipes with new cells are calculated via a combination of periodic structure theory and finite element method. Compared to the classical phononic crystal pipe, when a same lattice constant is adopted, the phononic crystal pipes with new cells obtain lower frequency band gaps or widen the low-frequency band gaps.

1. Introduction

Pipe systems are widely used in many fields. The pipe vibration may cause noise and malfunction. Low-frequency vibration control is a key technology in the vibration control of pipe systems. However, the limitations of pipe distribution and mass, the interaction between fluid and pipe wall, and the safety requirements make many low-frequency vibration control methods ineffective [1–3]. The new methods given by phononic crystals (PCs) theory may solve these problems [4–6]. The study of elastic wave propagation in phononic crystals has received increasing attention in the last several decades [7–14]. The presence of band gaps (BGs, also referred to as stop bands) in PCs, which blocks elastic wave propagation within the BG frequency range (BG frequency range, also referred to as BG bandwidth) [15], constitutes a new method to control noise and vibrations. The band gaps are introduced via two mechanisms: the Bragg-type band gap [16,17], which is due to the Bragg reflection, and the locally-resonant (LR) band gap [18], which is due to the scattering resonances stimulated by elastic waves of specific frequencies. The spatial modulation of the elasticity must be of the same order as the wavelength in the gap induced by the existing Bragg scattering mechanism [17]. For example, when the sound speed of steel is 3230 m/s, according to the formula of the central frequency of the BG ($f = c/2a$, c is the sound speed, a is the cell length), PC pipe made solely of steel must be used to control the vibration below 400 Hz, whose cell length is at least 4 m. Due to the size limitation in axial direction, it is difficult in practice to obtain low-frequency Bragg-type band gaps via using small size structure. The locally resonant BGs can exist in a frequency range two orders of magnitude lower than

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the one resulting from the Bragg scattering [17], which breaches the size constraint of Bragg scattering mechanism and may solve the difficult problem of obtaining low-frequency band gaps in a small size pipe structure. However, the locally resonant structure only solves the size limitation in axial direction. Besides, the bandwidth of LR band gap is usually narrow [19]. Although the effect of inertial amplification [20,21] can be used to improve the bandwidth of locally resonant band gap, it may result in additional mass and new conflicts in radial direction. Many pipe systems are arranged densely in practical applications. The size limitation in axial direction makes it impossible for us to have the operation. The Bragg-type PC pipe can completely solve the size limitations in radial direction. New manufacturing techniques such as 3D printing and new welding technology make it possible to process the Bragg-type PC pipes. Therefore, the key question is that can we get low-frequency band gaps via choosing the cell structure of Bragg-type phononic crystals for vibration control which is contradictive to the suggestion given by the existing related research. The suggestion given by the existing Bragg-type band gap mechanism is based on simple cell structures and material composition. So we shift our attention to the structural complexity and material composition of the cell and have found some new features. In this paper, the Bragg-type PC pipes with new cells are designed to study the influence of the cell structure and material composition on the band gaps. The periodic structure theory (PST) and the finite element method (FEM) are combined to perform the calculations for the new band structures. The results under a same lattice constant are then compared and analyzed to answer the question above.

2. New Bragg-type PC pipes

Fig. 1(a) shows the classical Bragg-type PC pipe composed of different materials. The materials are bonded only in axial direction. The dimension of one cell in the dashed line box marked as a is the lattice constant. The cell expansion along the radial direction and the three-dimensional mode are shown in Fig. 1(b). In order to study the feasibility of obtaining low-frequency Bragg-type band gaps by changing the cell structure under a same lattice constant, several new Bragg-type cells are designed in this paper. Each kind of the new cells contains two or more kinds of materials. Compared with the classical cell, the material binding degree is higher. The materials are bonded not only in axial direction, but also in radial direction. The pipe cell is cut into two halves in axial direction and several parts along circumferential direction at the same time. Different materials are alternately arranged along circumferential direction. Then, the two parts with different material in axial direction stagger an angle along circumferential direction. Fig. 2 shows the expansions of the cells cut into halves along circumferential direction and the corresponding three-dimensional models. Different colors represent different materials. One with the staggered angle of 180° shown in Fig. 2(a) is marked as two-part-180° cell and the other one with the staggered angle of 90° shown in Fig. 2(b) is marked as two-part-90° cell. Similarly, the cells cut into four equal parts in Fig. 3 are marked as four-part-90° cell and four-part-45° respectively. There are a lot of similar structures: three-part-120° or three-part-60°, six-part-60° or six-part-30°, and nine-part-40° or nine-part-20°, etc. In addition, a new cell also can be with a staggered distance in the axial direction. It is important to note that not all of the above cells will have a positive effect on the low-frequency BG. In the next calculation and analysis, the two relatively simple cells with positive effects including two-part-180° cell and four-part-90° cell are adopted to verify our assumptions.

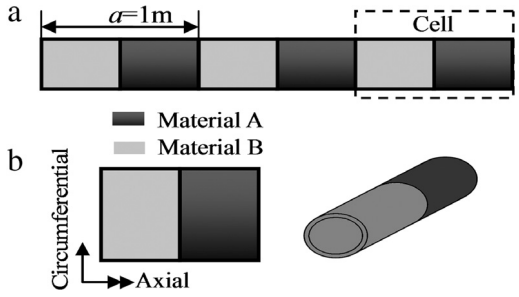

Fig. 1. The classical Bragg-type PC pipe composed of two kinds of materials.

3. Theoretical modeling and calculation

For classical PC pipe, the Euler beam model and Timoshenko beam model can be used to study the pipe vibration. However, for the Bragg-type PC pipes with new cells, since the materials are different at the same axial displacement, the beam models fail. The FEM used in the study of free wave propagation in periodic structures was first applied by Orris and Petyt [22]. Latter, similar approaches were developed by Mace and his team to predict dispersion relations of wave motion in structural waveguides [23]. The propagation constants and forced response can be computed by considering just one unit cell of the periodic system via combining periodic structure theory and FEM. Xiao et al., uses this method to study the periodic truss beams with members of different materials [24]. In our paper we use the same method to perform the calculations for the band structures of the new Bragg-type PC pipes. PST and FEM are combined by considering just one new Bragg-type cell for free wave propagation in Bragg-type PC pipe as follows. Without considering the impact of damping, the kinetic equation of the PC pipe cell is given by

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M})\boldsymbol{\delta} = \boldsymbol{F} \tag{1}$$

where, ω is angular frequency, $\boldsymbol{K}$ is stiffness matrix, $\boldsymbol{M}$ is mass matrix, $\boldsymbol{\delta}$ and $\boldsymbol{F}$ are node displacement and force. Combining the right (R) and left (L) boundaries, Eq. (1) can be written in expanded partitioned form as follows:

$$\begin{bmatrix} \boldsymbol{D}_{\mathrm{LL}} & \boldsymbol{D}_{\mathrm{LR}} \\ \boldsymbol{D}_{\mathrm{RL}} & \boldsymbol{D}_{\mathrm{RR}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\delta}_{\mathrm{L}} \\ \boldsymbol{\delta}_{\mathrm{R}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_{\mathrm{L}} \\ \boldsymbol{F}_{\mathrm{R}} \end{Bmatrix} \tag{2}$$

where, $\boldsymbol{D}$ is the dynamic stiffness matrix. Considering a free wave propagating along the infinite PC pipe, the Bloch theory [25] states that

$$\boldsymbol{\delta}_{\mathrm{R}} = e^{\boldsymbol{\lambda}} \boldsymbol{\delta}_{\mathrm{L}}, \qquad \boldsymbol{F}_{\mathrm{R}} = e^{\boldsymbol{\lambda}} \boldsymbol{F}_{\mathrm{L}} \tag{3}$$

where, $\boldsymbol{\lambda}$ is the propagation constant. It contains the information of Bloch vector and the lattice constant. Combining Eqs. (2) and (3), one obtains

$$\left.\begin{aligned} \boldsymbol{F}_{\mathrm{L}} &= (\boldsymbol{D}_{\mathrm{LL}} + e^{\boldsymbol{\lambda}} \boldsymbol{D}_{\mathrm{LR}}) \boldsymbol{\delta}_{\mathrm{L}} \\ \boldsymbol{F}_{\mathrm{L}} &= -(e^{-\boldsymbol{\lambda}} \boldsymbol{D}_{\mathrm{RL}} + \boldsymbol{D}_{\mathrm{RR}}) \boldsymbol{\delta}_{\mathrm{L}} \end{aligned}\right\}. \tag{4}$$

Then, one can derive the quadratic eigenvalue problem as follows:

$$\left[\boldsymbol{D}_{\mathrm{RL}} + (\boldsymbol{D}_{\mathrm{LL}} + \boldsymbol{D}_{\mathrm{RR}}) e^{\boldsymbol{\lambda}} + \boldsymbol{D}_{\mathrm{LR}} e^{2\boldsymbol{\lambda}}\right] \boldsymbol{\delta}_{\mathrm{L}} = 0. \tag{5}$$

The Eq. (5) can be reduced to the linear eigenvalue equation as follows:

$$\left(\begin{bmatrix} \boldsymbol{D}_{\mathrm{RL}} & \boldsymbol{D}_{\mathrm{RR}} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - e^{\boldsymbol{\lambda}} \begin{bmatrix} -\boldsymbol{D}_{\mathrm{LL}} & -\boldsymbol{D}_{\mathrm{LR}} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \right) \begin{Bmatrix} \boldsymbol{\delta}_{\mathrm{L}} \\ \boldsymbol{\delta}_{\mathrm{R}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix}. \tag{6}$$