

Tunable elastodynamic band gaps



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ABSTRACT

The effect of nonlinear elastic pre-stress on coupled compressional and vertically polarised shear elastic wave propagation in a two-dimensional periodic structure is investigated. The medium consists of cylindrical annuli embedded on a periodic lattice in a uniform host material. An identical inhomogeneous deformation is imposed in each annulus and the theory of small-on-large is used to find the incremental wave equations governing subsequent small-amplitude elastic waves. The plane-wave-expansion method is employed in order to determine the permissible eigenfrequencies. It is found that the application of pre-stress has a significant effect on the band structure, allowing stop bands to be controlled. The sensitivity of the choice of constitutive behaviour is studied and it is shown that the fundamental shear wave mode is largely unchanged for the class of strain energy functions considered here, whereas the compressional mode is considerably more sensitive to this choice.

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1. Introduction

Improvements in engineering and technology rely greatly on advanced, complex materials, which often possess intricate microstructure, permitting macroscopic behaviour that is not present in naturally occurring materials. Elastodynamic and acoustic phononic media present a broad range of opportunities for directing waves [1–3] and being able to design these materials carefully in order to enable wave focusing, filtering and directivity is greatly advantageous. Of specific importance is the notion of *tunable*, *configurable* or *re-configurable* phononic media, which have static and dynamic material properties that can be tuned in real time. Such materials have many obvious advantages over media with properties that are fixed upon manufacture. A standard approach is to modify the microstructure and then understand how this modification affects the macroscopic response on a static and dynamic level. A number of opportunities to enable this have been presented. Early work in tuning the *acoustic* response of such media focused on modifying the band gap properties of phononic media by rotating cylinders [4–6].

Several groups have considered using the influence of magnetic or electric fields as a means to perform external control over a crystal [7,8]. In [9], an electric field deforms annular cylinders that are embedded in air, piezoelectric materials are studied in [10–12] and electro-rheological materials in [13]. Thermal effects have also been proposed but the effect is usually fairly weak and so a phase transition effect is usually required [14,15]. These mechanisms for control provide the advantage of a *phononic switch* in such materials. Mechanical mechanisms to create *photonic* switches have also recently been considered, see e.g. [16,17].

It is well-known that nonlinear elastic pre-stress affects the propagation of subsequent linear elastic waves in the medium [18] and this effect has been exploited in order to modify the band-structure of elastodynamic phononic media. In [19,20] an inhomogeneous beam was considered and it was shown that the band structure can be manipulated effectively using pre-stress. It has recently been noted that the response of a nonlinearly elastic material can be extremely sensitive to the choice of constitutive behaviour [21].

An electrical bias has been employed in order to control deformation and hence band gaps in layered and fibre reinforced media [22,23]. Experiments in one dimensional structures were performed by [24]. By making use of numerical simulations, via the finite-element method, the mechanical tunability of three-dimensional structures has also recently been studied [25]. The effect of microstructural buckling of an elastic material was the

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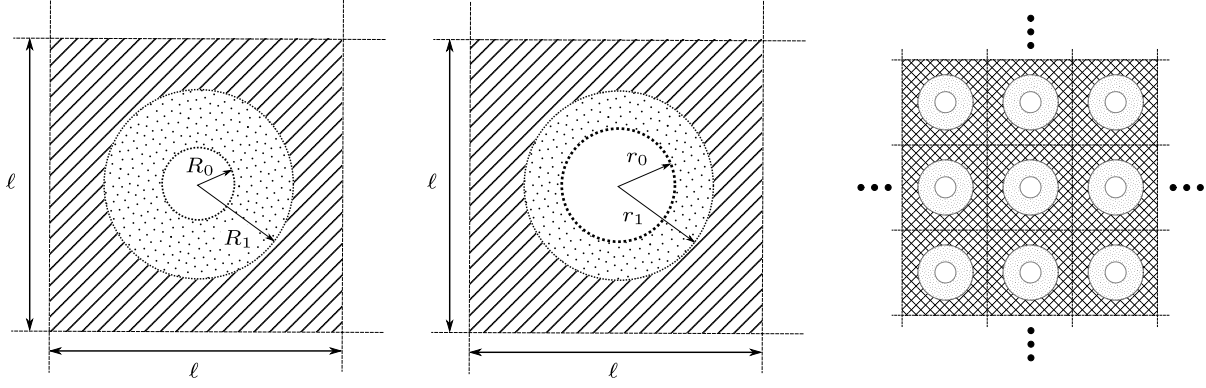


Fig. 1. Illustration of the unit cell for the undeformed configuration on the left and the deformed configuration in the middle. On the right hand side it is shown how the cylindrical annuli are embedded in a square periodic lattice with period ℓ in a stress-free, homogeneous medium shown by the hashed pattern.

focus of investigation in [26–29]. In [30] elastic microstructural helices were employed to control sound.

In this article a new mechanism for the control of band gaps in the full *elastodynamic* context is introduced, based on the work in [31], which was restricted to the scenario of antiplane elastic waves. In that work, incompressible hyperelastic annular cylinders were embedded periodically in an elastic medium and their deformation ensured that band gaps could be switched on and off. Further however, it was shown, based on the theory of hyperelastic cloaking [32–35] that some materials permit deformation in a manner such that in a normal setting from a geometric viewpoint they would possess band gaps but the pre-stress prohibits this; these materials were termed *phononic cloaks*. The theory of *small-on-large* was employed to derive the equations governing the propagation of superposed antiplane elastic waves and the plane-wave-expansion technique was extended in order to accommodate the inhomogeneous deformation present in the cylinders.

Here the same geometry as considered in [31] is employed, as depicted in Fig. 1 but here the annular cylinders are permitted to be *compressible* in order to accommodate the propagation of compressional waves.

2. Deformation and governing incremental wave equations

As illustrated in Fig. 1 the medium consists of a two-dimensional square array (with period ℓ) of nonlinear isotropic elastic annular cylinders each with initial inner and outer radii R_0 and R_1 respectively, density ρ_1 and linear elastic bulk and shear moduli κ_1 and μ_1 respectively, embedded inside a homogeneous elastic host medium of density ρ_0 and linear elastic bulk and shear moduli κ_0 and μ_0 . The unit cell is depicted in its stress-free state in Fig. 1(a). All annuli are assumed to be deformed identically. The deformation consists of an inflation (leading to an inhomogeneous radial deformation) and extension along its axis such that, crucially here, the outer radius of the cylinder remains unchanged, $r_1 = R_1$. This results in a deformed configuration as depicted in Fig. 1(b). In principal the host medium can be linear or nonlinearly elastic but here we shall consider it as linear. Subsequent elastodynamic wave propagation depends on the initial deformation and the governing equations are determined by appealing to the theory of small-on-large [18].

It is envisaged that the deformation results from inserting a stiff cylindrical inclusion with elastic properties μ_2 and κ_2 and density ρ_2 into the inner region of the annulus.

2.1. Annular cylinder inflation

The deformation of the annular cylinder is conveniently described in the form

$$R = R(r), \quad \Theta = \theta, \quad Z = z/\zeta \quad (1)$$

where the usual convention is followed, using upper (lower) case variables for the reference (deformed) configuration. Deformation is imposed such that

$$R_0 = R(r_0), \quad R_1 = R(r_1) = r_1. \quad (2)$$

Principal stretches are

$$\lambda_r = \frac{1}{R'(r)}, \quad \lambda_\theta = \frac{r}{R(r)}, \quad \lambda_z = \zeta. \quad (3)$$

A second order nonlinear ordinary differential equation (ODE) is determined from the first equation of static equilibrium $\text{div} \mathbf{T} = \mathbf{0}$, the second and third being trivially satisfied by the assumed form of deformation. The ODE takes the form

$$R'' = g(R, R', W, \zeta) \quad (4)$$

where W is the strain energy function governing the constitutive behaviour of the annulus and g is some function that is known only when W is prescribed. This ODE for $R(r)$ is subject to the boundary conditions (2).

In general the ODE (4) cannot be integrated analytically except for six classes of strain energy functions [36]. The issue with these materials types is that they are not considered physically realistic. Therefore two alternative strain energy functions shall be considered here that have been proposed as realistic models of compressible nonlinear elastic materials. First the form introduced by Levinson and Burgess [37] is employed:

$$W_{LB} = \frac{\mu_1}{2}(I_1 - 3) + \frac{\lambda_1 + \mu_1}{2}(I_3 - 1) - (\lambda_1 + 2\mu_1)(I_3^{1/2} - 1), \quad (5)$$

which was been proposed as a compressible extension of the incompressible neo-Hookean strain energy function and this shall be referred to here as LB. Note that $\lambda_1 = \kappa_1 - 2\mu_1/3$. The second form considered is a compressible extension of the Mooney–Rivlin strain energy function [38]

$$\begin{aligned} W_{CMR} = & \frac{\mu_1 S_1}{2} (I_1 - 3 - \log(I_3)) \\ & + \frac{\mu_1(1 - S_1)}{2} (I_2 - 3 - 2 \log(I_3)) \\ & + \frac{3\lambda_1 + 2\mu_1}{6} (I_3^{1/2} - 1)^2, \end{aligned} \quad (6)$$

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