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Coupling local resonance with Bragg band gaps in single-phase mechanical metamaterials

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ABSTRACT

Various strategies have been proposed in recent years in the field of mechanical metamaterials to widen band gaps emerging due to either Bragg scattering or to local resonance effects. One of these is to exploit coupled Bragg and local resonance band gaps. This effect has been theoretically studied and experimentally demonstrated in the past for two- and three-phase mechanical metamaterials, which are usually complicated in structure and suffer from the drawback of difficult practical implementation. To avoid this problem, we theoretically analyze for the first time a single-phase solid metamaterial with so-called quasi-resonant Bragg band gaps. We show evidence that the latter are achieved by obtaining an overlap of the Bragg band gap with local resonance modes of the *matrix* material, instead of the inclusion. This strategy appears to provide wide and stable band gaps with almost unchanged width and frequencies for varying inclusion dimensions. The conditions of existence of these band gaps are characterized in detail using metamaterial models. Wave attenuation mechanisms are also studied and transmission analysis confirms efficient wave filtering performance. Mechanical metamaterials with quasi-resonant Bragg band gaps may thus be used to guide the design of practically oriented metamaterials for a wide range of applications.

1. Introduction

Mechanical metamaterials are engineered periodic composites with exceptional dynamic properties. The possibility they provide to manipulate and attenuate elastic waves at various frequencies can be exploited for various applications, ranging from seismic shielding [1,2] or noise abatement [3] to subwavelength imaging [4] and thermal management [5]. These fundamental properties arise from metamaterial geometry and/or composition and are due to the existence of band gaps (BGs)—frequency ranges, in which wave propagation is inhibited. The frequencies and width of BGs depend on the contrast between mechanical properties of material phases and lattice parameters. Bragg BGs occur in Phononic Crystals (PCs) through destructive interference of waves scattered from periodic inhomogeneities at wavelengths comparable to the spatial periodicity of the lattice [6,7]. The resulting high operating frequencies make this type of structure unsuitable for noise mitigation or vibration isolation. Instead, hybridization BGs [8,9] are typically induced in metamaterials by resonant modes of the constituents, which interact with the wave field in the embedding medium [10]. These BGs are independent of the spatial configuration of the metamaterial and can be nucleated at much lower frequencies than Bragg BGs, but are usually rather narrow and require heavy resonators [10–14]. Thus, due to their complicated design or limited working performance [13,15,16], mechanical metamaterials are yet to become widespread in applications.

One promising solution to overcome these limitations is to exploit overlapping Bragg and local resonance BGs. The coexistence of both BG types in the same structure has already been demonstrated theoretically and experimentally for different systems at various frequencies [17–23], including in 3D sonic solid metamaterials with coated inclusions [24,25]. These studies

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highlight the BG formation mechanisms [9,17,22,23] and show that the coupling between the two BGs leads to the creation of a combined 'resonant Bragg' BG with a broad transmission gap in the sub-wavelength region [7,21,22,24,25]. These conclusions are also valid for surface guided waves [26] and acoustic waves in PCs with gas bubbles [27]. Recently, coupled resonant Bragg BGs have also been found in co-continuous metamaterials with enhanced mechanical properties [28]. All these studies involve composite metamaterial structures comprising at least two material phases, and the hybridization BG is usually associated with resonance modes of inclusions.

A more practical and promising solution are single-phase metamaterials, which have attracted increasing interest in the community [29–38], and in which both types of BGs have been shown to be present [39–43]. However, to the best of our knowledge, the conditions and physical mechanism of the coupling of Bragg BG with local resonances in single-phase structures has not yet been analyzed. In this work, we present evidence for single-phase 2-D and 3-D slab-type metamaterials with wide BGs due to the interaction between Bragg scattering and resonant modes. Moreover, we develop a new strategy to bring the BGs to overlap, using resonating modes of the matrix material instead of the inclusions, leading to so-called 'quasi-resonant Bragg' BGs. These BGs appear to exhibit efficient wave attenuation and stable frequency ranges for a wide range of geometric parameters of the inclusions. Thus, the proposed strategy shows promise for enhanced wave attenuation mechanisms coupled with the possibility of fabricating simpler structures, promoting the exploitation of mechanical metamaterials in real applications.

2. Two-dimensional mechanical metamaterials

2.1. Two-phase structures

First, we analyze 2-D phononic structures, in which pure transverse (out-of-plane) and mixed (in-plane) modes propagate independently, when the wave vector $\vec{k}$ is restricted to the XY plane. In this Section, we analyze in-plane modes only.

We start by considering a square array of circular steel inclusions (Fig. 1(a)) or cavities (Fig. 1(b)) in a typical polymeric material, such as that used in a 3-D printer, characterized by Young's modulus $E = 2$ GPa, Poisson's ratio $\nu = 0.4$ and mass density $\rho = 1050$ kg/m^3. The material parameters of isotropic steel are Young's modulus $E = 207$ GPa, Poisson's ratio $\nu = 0.3$, and mass density $\rho = 7784$ kg/m^3. As a first approximation, we neglect any dissipation losses. We consider the cavities to be filled by vacuum, so that no refraction of elastic waves occurs at their boundary. The radius of an inclusion or a cavity is R, and the distance between the centers of two neighboring non-diagonal inclusions or cavities is a. Fig. 1 shows the corresponding band structures for wave numbers varying along the boundary of the first Brillouin zone Γ–X–M evaluated by the Finite Element Method (FEM) for a representative unit cell with $a = 1$ mm and $R = 0.45\,a$. Blue and red curves indicate propagating and evanescent modes with real and imaginary values of the wave vector, respectively. The simulations are performed by applying Bloch periodic conditions at the unit cell boundaries and implemented using the commercial software COMSOL Multiphysics 4.3. The frequencies f are normalized as $\Omega = fa/c_t$, c_t being the transverse wave velocity. Directional and complete BGs are indicated by red and green shaded rectangles, respectively.

For the PC with steel inclusions (Fig. 1(a)), the wide complete band gap occurring between the third, $\Omega = 0.727$, and fourth, $\Omega = 2.071$, pass bands is due to Bragg scattering. Vibration forms at the BG bounds show localization of motion in the matrix material. The BG in the PC with cavities (Fig. 1(b)) is smaller in size and shifted to lower frequencies. The shift can be explained by the lower rigidity of the phononic structure, while the decrease of the BG size occurs due to the presence of localized modes, e.g. the fourth pass band, forming the upper BG bound. Although the vibration forms at the BG bounds shown in subfigures of Fig. 1 differ for the two considered structures, the BG in a PC with cavities is also due to the Bragg scattering mechanism. This can be deduced from the structure of the imaginary part of the spectrum, typical for Bragg BGs, which uniformly varies within the BG with a maximum value approximately at its mid-frequency [44], and from the similarity in the pass bands below the BGs.

Next, we analyze a metamaterial composed of polymeric circular cylinders with steel cores, which are connected by means of thin ligaments (schematically shown in the inset of Fig. 2). The radius of the steel inclusion is $R_{inc} = 0.25a$, the radius of the coated inclusion is $r = 0.35a$, and the thickness of the ligaments is $b = 0.05a$, where a is defined as previously. The corresponding band structure shown in Fig. 2 is characterized by three wide BGs separated by almost flat bands due to localized modes (the corresponding vibration patterns are shown in subfigures of Fig. 2). It is difficult to ascertain the physical nature of the BGs with certainty. On one hand, the imaginary part of the lowest BG resembles that expected for Bragg scattering (see e.g. Fig. 1); however, a localized mode occurs at the BG lower bound, when the coated steel cylinder vibrates as a rigid mass with the ligaments playing the role of springs. Below this mode, there is a pass band characterized by torsional motions of the coated cylinders, which are also typical for locally resonant materials [13]. Thus, the BG appears to be due to coupled Bragg and local resonance effects, as its large size also indicates. However, this metamaterial configuration is unpractical due to manufacturing difficulties and stability issues.

To stiffen the whole structure, we combine it with the previously considered PC with cavities. The resulting configuration is schematically shown in Fig. 3 together with the corresponding band diagram. This metamaterial is characterized by three BGs. The lowest BG bound is of the local resonance type and consists in a localized mode $\Omega = 0.182$ with the vibration pattern shown in Fig. 3, similar to that occurring in the BG in the coated cylinders lattice (see Fig. 2). The resonance nature of this BG is confirmed by the structure of the imaginary part of the diagram, typical for locally resonant metamaterials [45]. The third BG around $\Omega = 1.5$ is due to Bragg scattering, as indicated by the Bragg-type imaginary bands.

The second (widest) BG has a lower bound at the same frequencies as the Bragg BG of the PC with cavities, while its upper bound is at about twice that of the PC with cavities. The vibration pattern at the lower bound of the BG appears to be a combination of localized motions in thin ligaments (vibration pattern at $\Omega = 0.732$ in Fig. 2) and vibrations of the rhombus-shaped material portions originating from the PC with cavities. Therefore, at the lower bound of the second BG, the matrix sections oscillate as rigid bodies, while the coated inclusions are motionless. The localized motions in the matrix are similar to those in the inclusions also for torsional vibrations of rhombus-shaped matrix portions corresponding to the mode below the lower BG bound (vibration pattern at $\Omega = 0.532$ in Fig. 3). The described behavior is similar to local resonances at the lower bound of the lowest BG. However, the structure of the imaginary parts of the spectrum is totally different and shows enhanced wave attenuation typical for the coupled 'resonance-Bragg' BGs [7,24].

All the mentioned features suggest a new BG formation mechanism: the local resonances of the rhombus-shaped (matrix) parts are coupled with Bragg scattering in the same parts of the metamaterial. The presence of coated inclusions allows the localizing of the motions in the matrix and shifting the upper bound of the BG to higher frequencies by stiffening the whole structure.