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Design of cut unit geometry in hierarchical kirigami-based auxetic metamaterials for high stretchability and compressibility

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ABSTRACT

We studied the mechanical response of a recently developed new class of mechanical metamaterials based on the paper art of cutting, kirigami. Specially, the geometrical and structural design of representative cut units, via combined line cut, cut-out, and hierarchy of the structure, was explored for achieving both extreme stretchability and/or compressibility in kirigami metamaterials through experiments, alongside geometrical modeling and finite element simulations. The kirigami design was tested on constituent materials including non-stretchable copy papers and highly stretchable silicone rubber to explore the role of constituent material properties. The cut unit in the shape of solid rectangles with the square shape as a special case was demonstrated for achieving the extreme stretchability via rigid rotation of cut units. We found that compared to the square cut units, the theoretically predicted maximum stretchability via unit rotation in rectangle units (aspect ratio 2:1) increased dramatically from about 41% to 124% for the level 1 cut structure without hierarchy, and from about 62% to 156% for the level 2 hierarchical cut structure, which was validated by both experiments and simulations. To demonstrate the achievement of both extreme stretchability and compressibility, we replaced the solid square cut units with porous squares and re-entrant lattice shapes in silicone rubber based metamaterials. We found that a porous structure can enable an extreme compressibility of as high as 81%.

1. Introduction

Mechanical metamaterials [1] are attracting increasing interest in scientific research and engineering innovation due to their unprecedented physical properties [2–7], arising from the geometrical arrangement of their periodic unit cells. Upon deformation, mechanical metamaterials can be reconfigured beyond their original designs [8–14], offering an enhanced flexibility in performance by coupling dynamically changing structural configurations with tunable physical properties [15].

Very recently, kirigami ("kiri" means cut), art of paper cutting, is becoming an emerging research frontier due to its broad potential applications in design of pluripotent materials [16, 17], stretchable and conformable electronics [18], stretchable energy storage devices [19], optical tracking in solar cells [20], acoustic filters [21], and 3-D mesostructures fabrication [22]. By introducing hierarchical line cuts into thin sheets of elastomer, we and other researchers [18,23] showed that hierarchical cuts can generate highly stretchable, super-conformable, and ultra-soft reconfigurable metamaterials [18,23], which exhibit highly nonlinear stress–strain behaviors resulting from the hierarchically cut structure rather than their constituent material [23]. The underlying mechanism lies in the fact that stretching deformation in the cut structure is largely accommodated by the rigid rotation of cut units, rather than severely deforming the cut units themselves [18,23].

Despite the recent advancement, limitations and challenges remain. First, since the extraordinary properties in the cut-based metamaterials are mainly determined by the rotation of cut units [18,23], the order of rotational symmetry in the geometry of cut units will play an important role in determining the structural configuration governed mechanical behaviors of hierarchical kirigami metamaterials, including the extreme stretchability and tunable Poisson's ratio, which remain to be exploited. Second, the line cuts can only generate stretchable metamaterials [18,23]. The non-compressibility beyond their original cut configurations will largely hinder their potential applications in stretchable electronics [9] and other conditions, where both stretchability and compressibility are often experienced and required for functionality.

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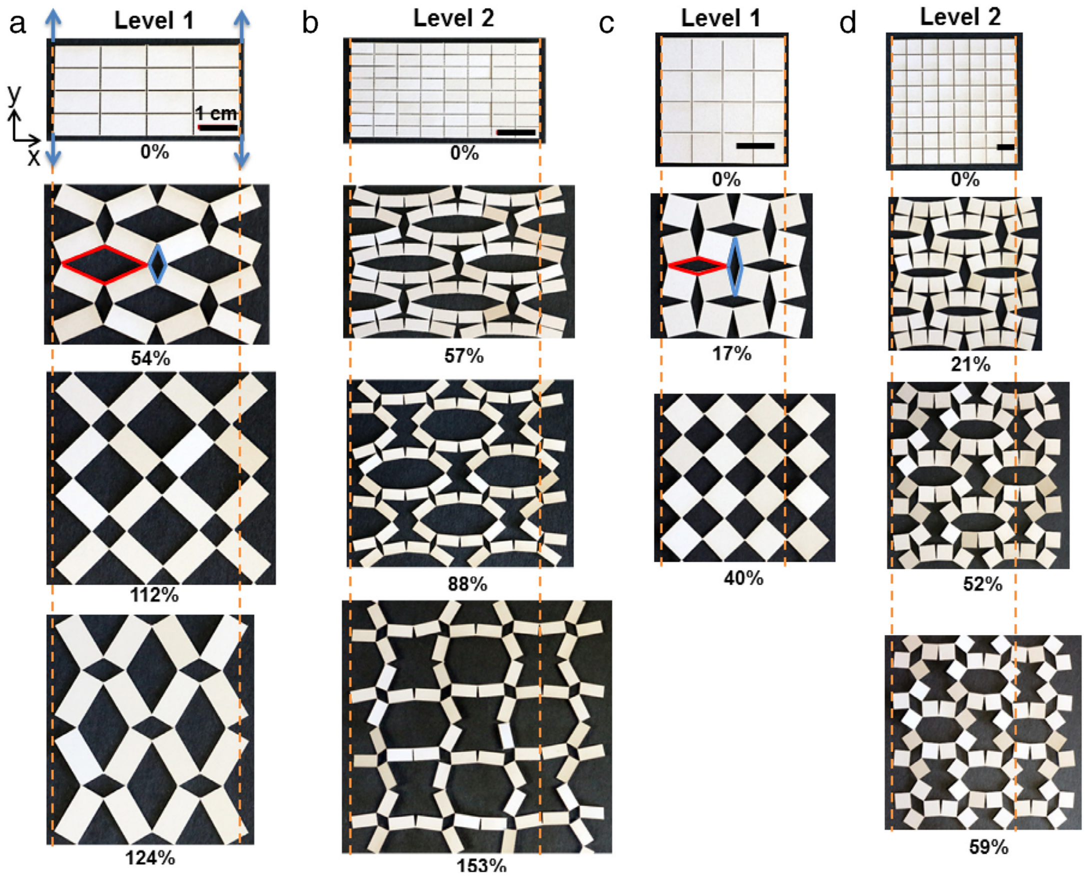

Fig. 1. Evolution of structural reconfiguration in hierarchical paper kirigami metamaterials constructed from rectangle (a–b) and square (c–d) cut units with the stretching along *y*-axis. (a) and (c): level 1 structure; (b) and (d): level 2 structure. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Here, we first explore the design of a more generalized rectangular cut unit in hierarchical metamaterials for achieving the extreme stretchability through rigid rotation of cut units (Fig. 1(a)–(b)), where square cut units [18,23] will become a special case in this work. We find that compared to square units, rectangle cut units can significantly enhance their extreme stretchability by more than three folds (for the case of aspect ratio 2:1) depending on the shape anisotropy of rectangles, as well as greatly extend the stretching strain range for exhibited auxetic behavior. Furthermore, we study the combined line cuts and cut-outs to circumvent the limitation of non-compressibility in cut-based stretchable metamaterials. We find that in addition to the stretchability provided by line cuts, the cut-outs enable the compressibility upon compacting the cut-outs via buckling, allowing the realization of both stretchability and compressibility in one structure at the same time.

2. Stretchable hierarchical paper kirigami metamaterials: rectangular vs. square cut unit

Fig. 1(a) shows a level 1 kirigami metamaterial consisting of a thin sheet of copy paper with prescribed patterned cuts *via* a laser cutter, where orthogonal cuts divide a rectangle unit cell into 4 connected sub-squares and the aspect ratio of the rectangle m, i.e. the ratio of its length a to width b, is set to be 2. A hierarchical structure with self-similarity can be constructed by repeating the same cuts in each sub-level of the rectangular units, leading to a level 2 (Fig. 1(b)) and higher-level hierarchical structure. Similarly, when the aspect ratio m is set to be 1, i.e. $m = 1$, the rectangle shape reduces to square, generating a hierarchical square-cut unit based kirigami metamaterial (Fig. 1(c)–(d)).

As demonstrated in Fig. 1, stretching the kirigami structures vertically led to the lateral expansion, exhibiting an auxetic behavior with a negative Poisson's ratio. Upon stretching, each cut unit rotated around its hinges, i.e. the cut tip on its corner, to generate expandable structures. Upon stretching the level 1 structure, the line cuts in rectangle cut units evolved into orthogonal rhombus pores with both small (blue) and large sizes (red) (2nd row of Fig. 1(a)) whereas uniform pores in square cut units (2nd row of Fig. 1(c)). Both structures expanded laterally through the rotation of cut units. With further stretching, a polarization orientation switch from x to y axis was observed in the distribution of rhombus pores for rectangular cuts (4th row of Fig. 1(a)) but not in the case of square cuts (3rd row of Fig. 1(c)). Here the orientation is defined as the direction along the long diagonal of a large rhombus. A more complex evolution of pore shapes and sizes with the stretching strain was observed in level 2 structures (Fig. 1(b) and (d)).

Meanwhile, hierarchical cuts can make a non-stretchable and non-shearable paper sheet become highly stretchable through rigid rotation of cut units. The last row in each column of Fig. 1 shows the expanded structural configuration right before failure, i.e. the rupture of hinges. Fig. 2 summarizes the failure strain for both square and rectangle cut units based hierarchical metamaterials. It shows that either increasing the hierarchical level of cut structures or increasing the aspect ratio of cut units leads to an enhanced maximum stretchability. However, compared to increase the hierarchical level, increasing the aspect ratio m of cut units can dramatically enhance the maximum stretchability. When m was increased from 1 to 2, we observed an over three folds increase in the maximum stretchability in level 1 structure from