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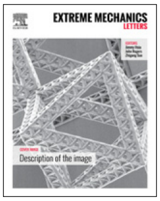

# Simulations of thermoelastic triangular cell lattices with bonded joints by finite element analysis

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ABSTRACT

Thermoelastic triangular cell lattices composed of bi-material curved ribs were designed and analyzed by finite element simulation. Positive, negative, or zero thermal expansion was possible by varying rib curvature if joints can pivot freely, as expected. Welded or bonded joints result in nonzero expansion but smaller in magnitude than that of a constituent material having higher thermal expansion coefficient. The effects of rib curvature variation for bonded joints were found to be negligible. Rib slenderness for both joints did not influence the coefficient of thermal expansion. We present a square lattice with bonded joints that has zero net thermal expansion; each curved bi-material rib has zero expansion.

## 1. Introduction

Various engineering structures in fields such as aerospace, civil engineering, and microelectronics often undergo large temperature changes [1]. They lead to thermal stresses caused by different thermal expansions in components of a structure which are made of materials with dissimilar coefficients of thermal expansion. In the field of aerospace, supersonic and hypersonic vehicles exhibit significant thermal stresses, and thermodynamic propulsion plants also experience similar stresses [2]. For these structures, dimensional stability (*i.e.*, structural integrity) is surely a key consideration. An example of the importance of dimensionally stable design of structures is the Hubble space telescope; considerable thermal distortions were produced by rapid temperature changes during its orbit, which led to undesired vibration of the telescope and the arrays [3]. A material's coefficient of thermal expansion is thus clearly one of the driving factors when selecting materials for structures subject to large fluctuations in temperature [4].

Materials of zero or minimal thermal expansion can provide dimensionally stable designs of structures subject to large temperature change. For example, zero or nearly zero thermal expansion is desirable in fields in which precise positioning of parts is critical, such as optics and electronics [5]. Similarly, aerospace and civil engineering applications, like piping systems designed with tight dimensional tolerances, are required to have minimal or zero thermal expansion for achieving dimensional stability under extreme variation in temperature [6]. By contrast, materials exhibiting negative thermal expansion are of interest for applications need to contract with increases in temperature [7]. It is also possible to control any desired thermal expansions (such as zero, or large positive, or large negative) in composite materials with void spaces by tuning design of their microstructure [8,9].

Recently, Lehman and Lakes [10] showed that zero coefficient of thermal expansion in a lattice made of bi-material curved rib elements can be achieved by designing hierarchical material structures with carefully chosen geometry and materials while optimizing the total mechanical stiffness of a thermoelastic triangular lattice, as shown in Fig. 1. Two different metals with different positive thermal expansion coefficients were used to design this microstructure, which is composed of ribs whose cross section was rectangular. The difference in thermal expansion coefficients leads to bending of the rib during temperature change, which results in a decrease of the distance between its ends which is exactly counterbalanced by overall thermal expansion of the rib. By carefully tuning geometric parameters of the bi-material curved rib, zero net thermal expansion in a honeycomb or lattice structure can be achieved. An overall thermal expansion coefficient for an individual bi-material curved rib element with pin-ended joints, as depicted in Fig. 2, is provided by Eq. (1) [10]. The derivation of this equation was based on Timoshenko's work for bi-material

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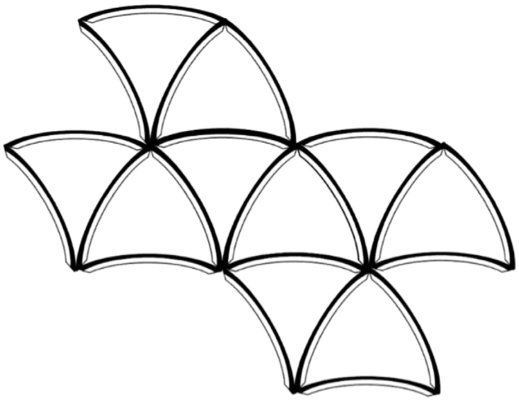

**Fig. 1.** The equilateral triangular lattice composed of bi-material curved ribs [10]. Material one with a lower coefficient of thermal expansion is shown as white (on the inner portion of each rib), while material two with a higher coefficient of thermal expansion is shown as black (on the other portion of each rib).

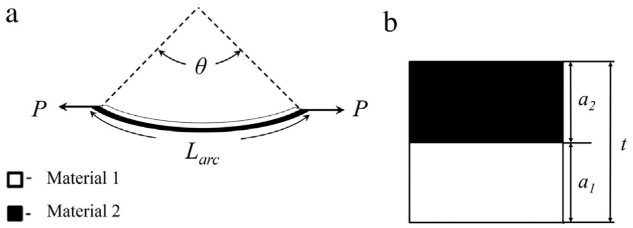

**Fig. 2.** (a) The loading state used to determine analytical results of zero coefficient of thermal expansion on the bi-material curved rib for pin-ended joints [10]. (b) $L_{arc}$ is the arc length of the rib, $\theta$ is the included angle, and $P$ represents axially applied load. The cross-sectional area of the bi-material curved rib [10] is shown.

**Table 1**
Material properties of selected materials.

| | Material 1 (invar) | Material 2 (steel) |
|---|---|---|
| Elastic modulus, $E$ | 140 GPa | 200 GPa |
| CTE, $\alpha$ | 1 µstrain/K | 12 µstrain/K |
| Poissons ratio, $\nu_{LT}$ | 0.28 | 0.3 |

strips [11]

$$\alpha_{net} = (\alpha_1-\alpha_2)\frac{L_{arc}}{t}\left(\frac{\theta}{12}\right)\frac{6(1+m^2)}{3(1+m)^2+(1+mn)\left(m^2+\frac{1}{mn}\right)} + \frac{\alpha_1+\alpha_2}{2} + (\alpha_2-\alpha_1)\left[\frac{4m^2+3m+\frac{1}{mn}}{nm^3+4m^2+6m+\frac{1}{mn}+4}-\frac{1}{2}\right] \quad (1)$$

where $\alpha_1$ and $\alpha_2$ are the thermal expansion coefficients of material one and two, respectively, $E_1$ and $E_2$ are the elastic modulus of material one and two, respectively, $L_{arc}$ is the arc length of the curved rib, $m$ is the thickness ratio of material one to material two (*i.e.*, $m = \frac{a_1}{a_2}$), $n$ is the elastic modulus ratio of material one to material two (*i.e.*, $n = \frac{E_1}{E_2}$), $\theta$ is the included angle and $t$ is the total thickness of the rib. In order to obtain zero coefficient of thermal expansion, material one, positioned on the inner portion of the curved rib, is required to have a smaller thermal expansion coefficient than material two on the outer portion. Invar was used as material one, while material two was steel, and their material properties are given in Table 1.

The analysis of such a lattice [10] relies on pin-ended joints between the bi-material curved ribs. These joints allow a force transmission to adjacent ribs while rotation is unconstrained; the ends of adjoining ribs are free to rotate with respect to one another. For these reasons, the curvature, and hence the moment, throughout a rib is uniform, with the result that it was straightforward to obtain the analytic solution given in Eq. (1). For other engineering reasons such as limitations during the manufacturing process, bonded (or welded) joints between these ribs may be desirable or may be necessary. For example, we are currently investigating the use of 3-D printing to fabricate new materials, and this technology will most likely require bonded joints. With bonded joints, the curvature and moment will not be uniform and the resulting differential equations will be difficult or impossible to solve in closed form. Hence, the use of finite element analysis is effective.

In the present manuscript, an individual bi-material curved rib connected by pin-ended joints was modeled using the commercial finite element program ANSYS APDL to verify the analytical results of Ref. [10]. A finite element model of the bi-material curved rib with bonded joints was then created and studied to determine if it could achieve zero or reduced thermal expansion. For both ribs, the effective coefficients of thermal expansion were calculated. The effects of rib curvature and of rib slenderness for both joints were also investigated. To incorporate the effects of interactions between adjacent ribs in a lattice, several finite element models of a thermoelastic triangular lattice were created by assembling the finite element models representing each of individual bi-material curved ribs. The influence of bonded joints between such ribs in the lattice was then investigated subject to uniform temperature change. The effective thermal expansion coefficient of the lattice was also computed. A change in the orientation of some of the ribs in the lattice was studied to observe how such a change would influence the overall thermal expansion. Finally, we conclude this paper with discussion of a square lattice with bonded joints that has zero net thermal expansion provided that each rib has zero thermal expansion.

## 2. An individual bi-material curved rib

### 2.1. Determination of optimized geometric parameters

In order to achieve zero thermal expansion coefficient of an individual bi-material curved rib, geometric parameters and material properties need to be specified carefully [10]. With Eq. (1) describing the net thermal expansion coefficient of the rib in terms of geometric parameters and elastic moduli of the two materials, the included angle to obtain a zero net thermal expansion coefficient can be numerically computed by varying the invar fraction, as illustrated in Fig. 3. Invar fraction is the ratio of the thickness of invar to the total thickness of the rib, and the rib aspect ratio, *AR*, is the ratio of the arc length to the total thickness of the rib. Each curve denotes a different aspect ratio for representing slenderness of the rib. Materials used in this graph are typical invar and steel. The prior analytical results described an optimum invar fraction as approximately 45% [10]. The input to FEM requires a more precise value than is needed to draw graphs for an analytical result. This value was extracted from Eq. (1) to show that the optimum fraction was actually 46.39% regardless of the rib aspect ratio. This parameter was confirmed by obtaining desired thermal expansion of a rib with pin-ended joints by finite element analyses.

The optimum invar fraction of 46.39% was then substituted into Eq. (1) to obtain optimized geometric parameters of an individual bi-material curved rib. With the total thickness of the rib of 1 mm and the rib aspect ratio of 10, the included angle was calculated as 0.4909 rad, the radius of curvature was found to be 20.3684 mm, and the thickness of materials one and two was computed as 0.4639 mm and 0.5361 mm, respectively. These optimized values are summarized in Table 2.