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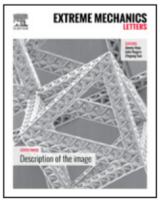

Buckling shape transition of an embedded thin elastic rod after failure of surrounding elastic medium

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ABSTRACT

When the compressive load to a thin elastic rod embedded in an elastic medium exceeds a threshold, the thin rod buckles into an exponentially decaying short wavelength profile to minimize the total energy of the system. As the compressive load continues to increase, the buckling amplitude increases correspondingly, until the rod/medium interface fractures and the short wavelength buckling profile morphs into a different shape as fracture propagates into the surrounding medium. In this study, such shape transition in the presence of surrounding medium failure is investigated using a combined experimental and theoretical approach. We identify the ansatz that can be used to describe the post-fracture buckling profile, and then develop a forward scheme using the energy principle to predict the buckling profile of the thin rod when fracture happens in the medium. We also develop a backward scheme where we use the post-fracture buckling profile to estimate the buckling profile before fracture of the surrounding medium. Comparison of experimental and theoretical results indicates that the modeling framework can be used to characterize the buckling profile transition of a thin elastic rod embedded in a fractured elastic medium.

1. Introduction

Traditionally, buckling of thin rods has been mostly considered as a failure mechanism to be avoided in engineering design. Recently, however, it has gradually been revealed that, as a form of mechanical instability, buckling plays a key role in the morphology generation of many natural systems [1–17]. Examples include the morphology of plant roots [1,2], oil pipes [3–5], elastic and viscoelastic films and shells [6–8], cytoskeletal microtubules in living cells [9–12], thin rods embedded in matrix [13–15] or granular media [16], and tortuous arteries [17]. For engineering structures, buckling and post-buckling behavior of infinite beams on nonlinear elastic foundations has also been studied using Koiter's improved theory [18]. Thus buckling phenomena have been investigated extensively to understand the fundamental mechanics, as well as to design synthetic systems taking advantage of this mechanical instability.

A thin elastic rod under axial compression tends to deform into a shape corresponding to its lowest deformation energy state. At low axial compressive forces, the rod experiences pure axial compression. Exceeding a critical compression threshold, the rod buckles with a certain mode number n that depends on the bending rigidity of the rod and the elastic properties of the medium where the thin rod is placed. In media with negligible elastic modulus such as air, the rod tends to buckle with the longest possible wavelength, which is on the same order as the rod length, with a pre-factor depending on the boundary conditions. However, when embedded in an elastic medium with a finite elastic modulus, the buckling wavelength decreases and the critical buckling force threshold increases [9]. Specifically, this short wavelength buckling mechanism may hold the key to understanding how microtubules can strengthen the structure of living cells [9–12].

As observed experimentally, there is a decay length in the buckling profile of microtubules within cells when the microtubules interact with cell periphery [9]. Das et al. predicted a decay length of buckling profile of a thin elastic rod embedded in an elastic medium with nonlinear elastic behavior [10]. Shan et al. modeled buckling of a superelastic rod in a biopolymer matrix using a similar theoretical framework, and showed experimentally that the nonlinear elastic behavior of the medium is outweighed by the linear responses, as predicted by modeling results [13]. To our best knowledge, however, all previous works have only investigated the buckling phenomenon assuming small deformations in the surrounding media, excluding the possibility of any failure of the embedding medium. For a thin elastic rod embedded in a biopolymeric matrix, as the amplitude of the short wavelength buckling increases, the deformation of the surrounding media in the vicinity

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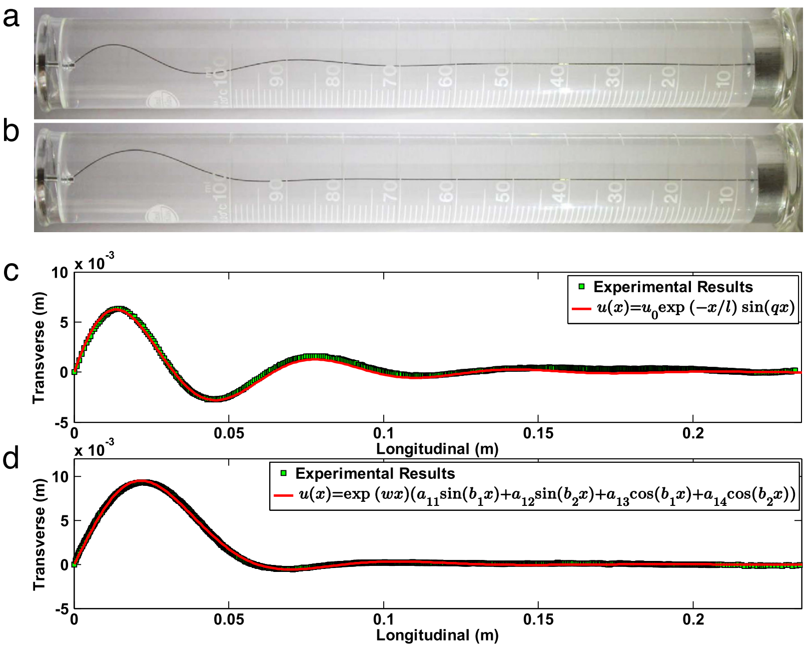

Fig. 1. (a) Buckling shape of a compressed wire embedded in gelatin before fracture. (b) Buckling shape of a compressed wire embedded in gelatin after fracture. (c) The deformation profile of the compressed wire in panel (a) and a fitted curve by Matlab. (d) The deformation profile of the compressed wire in panel (b) and a fitted curve by Matlab.

of the rod also increases. At certain point, the distinct interface between the rod and the polymeric medium will fail. Then the crack will propagate into the elastic medium and finally stops at a position that a new balance is reached (Fig. 1). This failure behavior may not happen in nature such as microtubules buckling during interaction with cell periphery, since the cytoplasm in living cells is more viscoelastic than gelatin, but it is critical to understand the mechanical behavior of engineered biomimetic systems where fracture in the medium can occur.

In this paper, we show experimentally the shape transition of the buckling profile of a biopolymer-reinforced rod when the surrounding matrix fractures. We identify a model that can be used to describe the post-fracture buckling profile. We then present both forward and backward schemes to predict the short wavelength buckling shape transition. For forward buckling shape transition, we treat the buckling profile as a 2D crack front and use crack propagation criteria to determine the buckling profile of the rod after failure of the medium, taking the buckling profile right before fracture as input. For the backward scheme, our method balances the deformation energy of the rod and the medium before and after the failure of the matrix to estimate the short wavelength buckling profile before fracture, taking the post-fracture profile as input. We also conduct a series of experiments featuring surrounding media with different elastic moduli to verify the applicability of our modeling framework. By providing new knowledge of the fundamental mechanics, these results will help understand buckling of microtubules in single cells, and shed light on new design principles for biologically inspired materials.

2. Materials and methods

2.1. Experiments

To experimentally investigate the buckling profile of a thin elastic rod in a biopolymer matrix, we use superelastic nitinol wires (55% nickel, 45% titanium) with a diameter of 203.2 μm obtained from Small Parts, Inc., which were straightened and annealed by the manufacturer per ASTM F2063. We assessed the Young's modulus, E, of the wires at room temperature to be $E = 60.8 \pm 1.0$ GPa [13]. We use porcine gelatin (Sigma Aldrich) with a Bloom value of 300 and concentrations of 20 g/l, 25 g/l and 30 g/l for the embedding matrix. Bloom value is a standard industrial measure to assess gelatin quality and is associated with the shear modulus of gelatin gels. Shear moduli of gelatin media used in this study were estimated to be $G = 0.64$ kPa, $G = 0.87$ kPa and $G = 1.12$ kPa for concentrations of 20 g/l, 25 g/l and 30 g/l, respectively [13,19]. We inserted nitinol wires with a length of 22.5 cm into graduated cylinders filled with liquid gelatin solutions. Then we put the samples in a fridge at a temperature of 5 °C for 24 h. After that the gelatin solution solidified and the nitinol wire was embedded inside biopolymer matrices, aligned along the axis of the graduated cylinder. The samples were exposed to room temperature at 24 °C for 5 h before testing.

The compressive force was exerted from one end of the wire using an adapter to simulate the hinged boundary condition while the other end of the wire was hinged with another adapter that was placed at the bottom of the graduated cylinder (Fig. 2). We increased the compressive loading quasi-statically using an ElectroForce® 3330 test instrument until we got close to the critical values for failure observed previously by Shan et al. [13]. We then increased the displacement step by step with a step size of 0.2 mm and a constant displacement rate of 0.1 mm/s within each step. We did this because it is well known that delayed fracture is common for soft and brittle gels [20]. Between the steps we held the displacement still for 10 s to observe whether failure occurred or not. If not, we further increased the loading until the abrupt shape transition was observed. Digital images of the rod deflections before and after the gelatin failure were taken by a Canon HD digital camera (EOS Rebel T5i). An image analysis code written in Matlab was used to analyze the images of rod deflections.

2.2. Theory and modeling

It has been reported in the previous studies [10,13] and also observed in the current work that the rod buckles in a plane and does not show any out-of-plane rotations for both of the pre- and post-fracture cases, provided that the rod in the graduated cylinder is aligned approximately in an axisymmetric manner. The