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On the mechanics of tetrakis-like lattices in the stretch-dominated regime

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ABSTRACT

We derive general conditions for the design of two-dimensional *stiffest elastic networks* with tetrakis-like (or 'Union Jack'-like) topology. Upon generalizing recent results for tetrakis structures composed of two different rod geometries (length and cross-sectional area), we derive the elasticity tensor of a lattice with generalized tetrakis architecture, which is composed of three kinds of rods and generally exhibits anisotropic response. This study is accompanied by an experimental verification of the theoretical prediction for the longitudinal modulus of the lattice. In addition, the introduction of a third rod geometry allows to extend considerably the possible lattice geometries for isotropic, stiffest elastic lattices with tetrakis-like topology. The potential of the analyzed structures as innovative metamaterials featuring extremely high elastic moduli vs. density ratios is highlighted.

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1. Introduction

Recently, there has been a growing interest in the design and fabrication of lattice metamaterials exhibiting a variety of 'extreme' behaviors not found in natural materials. These may include: exceptional strength- and stiffness-to-weight ratios; excellent strain recoverability; very soft and/or very stiff deformation modes; auxetic behavior; phononic band-gaps; sound control ability; negative effective mass density; negative effective stiffness; negative effective refraction index; superlens behavior; and/or localized confined waves, to name some examples (refer, e.g., to [1–10] and references therein).

As a matter of fact, a challenging approach to fill holes in material property charts (relating elastic stiffness and/or strength properties to material density) consists of playing with the microstructure of lattice materials in order to obtain an optimal combination of material and space (voids) at different scales [1]. Lattice metamaterials are structural networks made up of a large number of unit cells, which feature macroscopic length scales much larger than the length scales of the individual rods, and are such that their mesoscopic mechanical properties mainly derive by the geometry of the microstructure, rather than from the chemical composition of the material. Lightweight and strong lattices with nanoscale features and hierarchical architecture have been recently fabricated through the coating of additively manufactured polymeric scaffolds with metallic or ceramic materials, obtaining ultralight hollow-tube ceramic nanolattices that exhibit ultrastiff properties across more than three orders of magnitude in density [8], and/or ductile-like deformation and recoverability [6]. Attention is increasingly being given to metamaterials that feature geometrical nonlinear behavior, and precompression-tuned response [11–14].

In a recent work, Gurtner and Durand [15] studied the mechanical properties of isotropic networks of elastic rods in the linear elastic regime. As long as the typical dimensions of a junction are the same as the typical rod thickness, the energy cost associated with node deformation can be neglected in comparison with the rod stretching energy. However, no assumption is made on the relative importance of energy cost associated with node deformation and rod bending, so the mechanical response is generally not equivalent to those of pin-jointed structures. On dimensional grounds [1], it is clear that networks deforming primarily through the beam stretching mode are much stiffer than those deforming through the bending mode. However, stiffness still varies significantly among stretch-dominated networks. Only few structures have the peculiarity of deforming through beam stretching rather than bending: most structures will indeed deform primarily through other mechanisms than pure beam stretching. As an illustration, an hexagonal network will deform through beam bending

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mode if the energetic cost of node deformation is relatively much higher, and through node deformation in the opposite limit (which coincides to pin-jointed structures). Gurtner and Durand [15] have demonstrated the existence of *stiffest elastic networks*, which are stiffer than any lattice materials featuring the same symmetry, density and rod elastic properties. These stretch-dominated networks deform in *affine* way down to the heterogeneity scale *under any loading conditions* that are compatible with the linear elastic regime. Their elastic moduli constitute upper-bounds which are identical (3D) or below (2D) the well-known Hashin–Shtrikman (HS) bounds in the low-density limit. Then, these bounds are more precise than the HS bounds, but limited to networks of elastic rods only, while HS bounds apply to any diphasic structures. It is also worth noting that Deshpande et al. [16] have shown that triangulated structures having bars mutually clamped in the joints still exhibit stretching-dominated regime, and the collapse load is dictated mainly by the axial strength of the struts.

In the two-dimensional (2D) case, a special class of stiffest elastic networks is that of structures showing *tetrakis* (or '*Union Jack*') architecture, that is, lattices that tassellate the plane through square modules of right isosceles triangles [17]. By design, these lattices employ rods with two different lengths: one for the horizontal and vertical rods, and one for the diagonal rods. The cross-sectional areas are then adjusted to satisfy isotropic elastic properties [15].

From the fabrication point of view, both stretching-dominated and bending-dominated lattices can be fabricated employing additive-manufacturing technologies. Some examples are given in [18], in which mechanical microarchitected metamaterials made out of highly stretchable elastomers are fabricated through projection micro-stereo-lithography. Available literature results in this area confirm the theoretical findings about the stiffer response of stretch-dominated lattices structure, as compared to structures featuring relevant bending deformation effects at the nodes and within the bars [6,8]. It is noteworthy that the stretching-dominated response survives in cellular structures away from idealized networks with freely hinged joints [19]. The additive manufacturing of lattices featuring rods tapered near the junctions has also been investigated [20,21], with the aim of minimizing bending effects. The role played by mechanical interlocking connections has been studied in [22].

The present Letter presents a multifold generalization of the results obtained by Gurtner and Durand for tetrakis lattices [15]: (i) we derive the elasticity tensor of a tetrakis lattice with arbitrary shape and anisotropic response (Section 2); (ii) we present an experimental validation of the longitudinal elastic modulus predicted by such a theory against laboratory tests on a physical model (Section 2.2); (iii) we derive more general optimality conditions for the achievement of 2D stiffest networks (Section 3), which assume the presence of three different kinds of rods (horizontal, vertical, and diagonal) in the unit cell. The given results allow us to develop general conditions for the achievement of stiffest elastic networks in 2D, and pave the way to the design of stiff and lightweight structures featuring either one dimension much larger than the others (plane strain), or one dimension much smaller than the others (plane stress). These may be e.g. employed to design lightweight and stiff components of aeronautical structures, or next generation facades of tall buildings.

2. Anisotropic response of tetrakis-like lattices

Gurtner and Durand focus their study [15] primarily on stiffest elastic networks with isotropic symmetry (see also [23]). In the present work, we initially extend this study by analyzing the existence conditions for anisotropic structures with '*tetrakis-like*' architecture that deform affinely under any loading conditions. Such lattices tessellate the plane through rectangular –rather than square – modules of right triangles that show arbitrary aspect ratios between horizontal and vertical edges (Fig. 1(a)). Their elementary unit cell (or '*building block*') consists of the hatched region shown in Fig. 1(b), which features at least two axes of geometric symmetry (depending on the h_1 vs. h_2 ratio). The tetrakis lattices studied in [15] are obtained as a special case, by setting $h_1 = h_2$, assuming two different cross-sectional areas for the horizontal and vertical elements (first cross section) and the diagonal elements (second cross section), and using the same material for all the rods. We hereafter allow our tetrakis-like lattices to exhibit different materials and cross-section in different rods, and make use of the symbols A_k, L_k and E_k to respectively denote the cross sectional area, the reference length, and the Young modulus of the kth rod forming the building block shown in Fig. 1(b), which connects the central node 0 to node k ($k = 1, \ldots, 8$).

Following the work of [15], we look for the structural conditions under which a tetrakis-like architecture deforms affinely down to the microscopic scale, given an arbitrary, homogeneous and infinitesimal deformation of the lattice at the mesoscopic scale. In a first step, we calculate the strain energy that would be associated with such an affine deformation. We describe such a deformation through a displacement field of the form

$$\mathbf{u} = \boldsymbol{\varepsilon}\mathbf{x}, \tag{1}$$

where $\mathbf{x}$ denotes the position vector, and $\boldsymbol{\varepsilon}$ denotes the infinitesimal strain matrix with Cartesian components ε_{ij} with respect to a frame $\{O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ having the $\mathbf{e}_1$ and $\mathbf{e}_2$ unit vectors aligned with horizontal and vertical rods, respectively, and the $\mathbf{e}_3$ unit vector orthogonal to the lattice plane. It is an easy task to compute the strain energy $\mathcal{E}_{\mathcal{L}}$ associated to an affine deformation of a tetrakis-like lattice as follows

$$\begin{aligned}\mathcal{E}_{\mathcal{L}} = {} & \frac{\varepsilon_{11}^2}{2}\left(h_1(E_1A_1 + E_5A_5) + \frac{h_1^4}{16h_3^3}\right.\\ & \left.\times\ (E_2A_2 + E_4A_4 + E_6A_6 + E_8A_8)\right)\\ & + \frac{\varepsilon_{22}^2}{2}\left(h_2(E_3A_3 + E_7A_7) + \frac{h_2^4}{16h_3^3}\right.\\ & \left.\times\ (E_2A_2 + E_4A_4 + E_6A_6 + E_8A_8)\right)\\ & + (2\varepsilon_{12}^2 + \varepsilon_{11}\varepsilon_{22})\frac{h_1^2h_2^2}{16h_3^3}\,(E_2A_2 + E_4A_4 + E_6A_6 + E_8A_8)\\ & + \varepsilon_{12}\left(h_1^2\varepsilon_{11} + h_2^2\varepsilon_{22}\right)\frac{h_1h_2}{8h_3^3}\,(E_2A_2 - E_4A_4 + E_6A_6 - E_8A_8)\,.\end{aligned} \tag{2}$$

Let us define the solid volume intercepted by the building block as $V_{\mathcal{L}} = \sum_k A_k L_k$, and the solid volume fraction as $\phi = V_{\mathcal{L}}/V$, where V denotes the volume of the building block. The homogenized strain energy density of the lattice is computed as follows

$$\varphi_{\mathcal{L}} = \frac{\mathcal{E}_{\mathcal{L}}}{V} = \phi\frac{\mathcal{E}_{\mathcal{L}}}{V_{\mathcal{L}}}. \tag{3}$$

We now establish the structural properties of the lattice that are compatible with an affine deformation down to the microscopic scale, by enforcing the balance of forces everywhere in the structure. Under affine deformation, forces distributed in the lattice are parallel to the rods, and the balance equation at every node $k = 1, \ldots, 8$ yields:

$$E_iA_i\varepsilon_i = E_{i+4}A_{i+4}\varepsilon_{i+4} \quad i = \{1, \ldots, 4\}, \tag{4}$$

where $\varepsilon_i = \mathbf{e}_{0i}\boldsymbol{\varepsilon}\mathbf{e}_{0i}$ is the extension of rod connecting nodes 0 and i, and $\mathbf{e}_{0i}$ its unit tangent vector. Trivially, $\mathbf{e}_{0i+4} = -\mathbf{e}_{0i}$, $\varepsilon_{i+4} = \varepsilon_i$,

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