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Non-reciprocal flexural wave propagation in a modulated metabeam

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ABSTRACT

Flexural wave propagation in an Euler–Bernoulli beam coupled to a set of spring–mass resonators is investigated in the presence of a pump wave in the form of a space–time modulation of the beam-resonators coupling stiffness. A phase matching condition implies then that waves incident along or against the pump wave behave differently and gives rise in select frequency bands to one-way blocking and conversion of waves. In particular, one-way optical–acoustic transitions are proven possible and are quantified. Various orders of magnitude of relevant physical quantities, such as gap widths and interaction lengths, are estimated so as to guide future experimental implementations.

1. Introduction

Dynamic materials or spatio-temporal composites are materials whose properties change not only in space but also with time. Unlike smart structures that can adapt to slowly changing environments and loadings, the constitutive properties of dynamic materials vary at a rate comparable to the frequency of waves traveling through. In doing so, dynamic materials become a playground for completely new wave phenomena [1–3]. Recently, in the context of breaking time-reversal symmetry and reciprocity in linear non-lossy elastic media, a particular class of dynamic materials with properties modulated in space and in time in a periodic wave-like fashion have allowed to achieve new functionalities in selective and directional wave control including reversed Doppler effect [4,5], one-way mode conversion [6–8], unidirectional bandgaps [9,10] and broadband quasistatic unidirectional wave acceleration [1,11].

There is a variety of ways in which space–time wave-like modulations, so-called "pump waves", can be generated. Most require the mechanical system to be active or to be coupled to some active components. For instance, the elastic stiffness can be wave-like modulated by shedding a moving train of laser beams on a photoelastic medium [9,12] or by controlling the electric input of a stack of piezoelectric components [13–15]. Further, both mass density and bulk modulus can be controlled by appropriately distributing a magnetic field over a magnetorheological elastomer [16]. The pump wave can also be of a mechanical origin by following a "small-on-large" approach. In this scheme, a large-amplitude disturbance playing the role of a pump wave will effectively change the properties of the host medium by a non-linear mechanism (e.g., shock waves [4,5], or contact [17,18]) for any small-amplitude disturbance playing the role of the traveling wave.

In this letter, we investigate flexural wave propagation in a modulated metabeam (Fig. 1). In particular, we demonstrate, using asymptotic and numerical methods, a number of non-reciprocal effects including one-way conversion and blocking of waves. The use of a metabeam as a benchmark for these phenomena has a threefold motivation. First, a metabeam has a dispersive behavior accentuated by the resonance phenomenon. The resulting enriched dispersion diagram allows, under the influence of a pump wave, to observe several non-reciprocal effects simultaneously, a possibility that is precluded in the absence of dispersion and/or of optical branches [9,10]. Second, in the suggested benchmark, the beam-resonators coupling stiffness is more accessible for modulation in an experimental setting than what has been suggested elsewhere [7] as evidenced by the work of Casadei et al. [13] and Chen et al. [14,15]. Third, the use of resonators allows to bring to low frequencies scattering phenomena otherwise only observable at phononic frequencies and further permits to control the onset frequency of these phenomena in a way that is unaffected by geometrical parameters.

2. Theory

We begin by coupling the motion $u(x,t)$ of an Euler–Bernoulli beam with the motion $v(x,t)$ of a set of resonators as illustrated on Fig. 1 so that the governing equations read

$$G\partial_x^4 u + \rho\partial_t^2 u = \frac{1}{d}\widetilde{h}(v-u), \qquad m\partial_t^2 v = \widetilde{h}(u-v), \tag{1}$$

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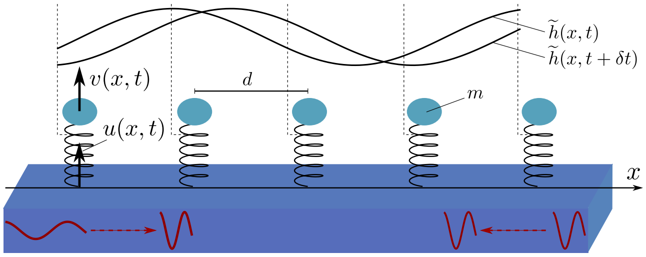

Fig. 1. Schematic of wave propagation in a modulated metabeam.

where G and ρ respectively are bending stiffness and mass density per unit length of the beam, m is the mass of a resonator, $\widetilde{h}$ is the modulated spring constant coupling the resonators to the beam and d is the spacing between two consecutive resonators. Although originally defined over a discrete set of locations where resonators are attached, $\widetilde{h}$ is assumed to be a function of the continuous variable x, which is a valid hypothesis in the long wavelength regime of interest in this letter. For later use, we introduce mass density per unit length of the resonators $\rho' = m/d$ and the normalized spring constant $\widetilde{k} = \widetilde{h}/d$. The modulation is wave like and has the form $\widetilde{h}(x,t) = h + 2\delta h \cos(q_m x - \omega_m t)$ where q_m and ω_m are the wavenumber and frequency of the modulation, respectively.

The modulation is assumed weak in the sense that the perturbation δk is small compared to the uniform offset k. Weak modulations are of interest as they offer control over unidirectional scattering and conversion phenomena in a way that is unmatched in strongly modulated media. The fact that the considered modulation is sinusoidal is of lesser importance and will not play a significant role in what follows. Weak sine-wave-like modulations of non-dispersive media have been studied in earlier works in the context of parametric amplification by many authors [19–23]. Here, we build on their work and extend their results to this case study where strong dispersion effects and multiple dispersion branches are at play.

In the absence of the modulation, a harmonic plane wave $u_0(x,t) = U_0 e^{i(q_0 x - \omega_0 t)}$ will propagate through the beam if it satisfies the dispersion relation $D(q_0, \omega_0) = 0$ with

$$D(q,\omega) = Gq^4 - \rho_{\text{eff}}(\omega)\omega^2, \qquad \rho_{\text{eff}}(\omega) = \rho + \frac{m/d}{1 - \omega^2/\Omega^2}, \tag{2}$$

where $\Omega = \sqrt{h/m}$ is the resonance frequency of the resonators. In the presence of the modulation, the incident wave u_0 will be scattered thus generating a second wave $u_j(x,t) = U_j e^{i(q_j x - \omega_j t)}$ whose wavenumber and frequency satisfy $D(q_j, \omega_j) = 0$ and are given thanks to Floquet–Bloch theorem by the phase matching condition

$$q_j = q_0 + j q_m, \qquad \omega_j = \omega_0 + j\omega_m, \tag{3}$$

where j is a non-zero integer. Waves u_0 and u_j are thus seen as two modes traveling in the non-modulated metabeam but coupled by the modulation: when one is incident, the other is scattered. Consequently, scattered modes are solutions to the equations $D(q_0,\omega_0) = D(q_j,\omega_j) = 0$, or thanks to the phase matching condition, $D(q_0,\omega_0) = D(q_0 + jq_m, \omega_0 + j\omega_m) = 0$. These solution modes can be determined graphically; see Fig. 2. Other non-solution modes are not coupled: when one is incident, no scattered wave is generated, at least in the context of the present leading order theory [24].

At this stage, the directional behavior of the metabeam can be anticipated. As a matter of fact, from Fig. 2, it is seen that when two modes (q_0,ω_0) and (q_j,ω_j) are coupled, modes $(-q_0,\omega_0)$ and $(-q_j,\omega_j)$ are not. Thus, a wave form scattered if incident to the left will not be scattered if incident to the right and vice versa.

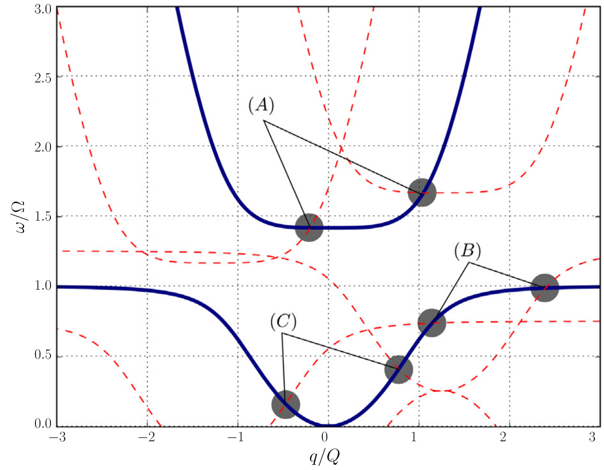

Fig. 2. The dispersion curve of the non-modulated metabeam (solid line) translated by $\pm(q_m, \omega_m)$ (dashed lines). The parameters of the modulation are $q_m = 1.25Q$ and $\omega_m = 0.25\ \Omega$. Pairs of intersection points labeled A, B and C correspond to pairs of coupled modes: when one is incident, the other is scattered. Here, the first legs of pairs A, B and C are given by $\omega_0^A = 1.41\Omega$, $q_0^A = -0.2Q$, $\omega_0^B = 0.73\ \Omega$, $q_0^B = 1.14Q$, $\omega_0^C = 0.15\ \Omega$ and $q_0^C = -0.48Q$ whereas the second ones are obtained by translation: $\omega_1^{A,B,C} = \omega_0^{A,B,C} + \omega_m$ and $q_1^{A,B,C} = q_0^{A,B,C} + q_m$.

When it occurs, scattering will modify the wavenumbers and frequencies of the traveling waves so that the state of the modulated beam to leading order becomes

$$u(x,t) = \left(U_0 e^{i(q_0 x - \omega_0 t)} + U_j e^{i(q_j x - \omega_j t)}\right) e^{i(\delta q x - \delta\omega t)} \tag{4}$$

where δq and $\delta\omega$ are first order corrections to $q_{0,j}$ and $\omega_{0,j}$. Their inverses $1/\delta q$ and $1/\delta\omega$ will define the characteristic space and time scales of the interaction between incident and scattered waves. In particular, $1/\delta q$ will be interpreted as the penetration depth of a blocked wave or the conversion distance of a transmitted wave (see Eqs. (7) and (9) below). Inserting this ansatz into the governing equations, we recover a couple of compatibility equations reading

$$\begin{bmatrix} 4q_0^3 G\Pi(\delta q - \delta\omega/c_0) & \rho'^2\omega_0^2\omega_j^2\overline{\delta_j k} \\ \rho'^2\omega_0^2\omega_j^2\delta_j k & 4q_j^3 G\Pi(\delta q - \delta\omega/c_j) \end{bmatrix} \begin{bmatrix} U_0 \\ U_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

where $\delta_j k$ is the jth Fourier component of the modulation, $\Pi = (k - \rho'\omega_0^2)(k - \rho'\omega_j^2)$ and $c_{0,j}$ is group velocity $-\partial D/\partial q \big/ \partial D/\partial\omega(q_{0,j}, \omega_{0,j})$.

3. Results

In our context, that of a modulation with a unique Fourier component, it is enough to consider $j = \pm 1$ since otherwise $\delta_j k$ vanishes but the results generalize immediately to modulations with multiple Fourier components. By setting the determinant of the above system to zero, the corrections $(\delta q, \delta\omega)$ can be determined for each pair of coupled modes $(q_{0,j}, \omega_{0,j})$; see Fig. 3(I-a,b). The resulting dispersion curve is shown on Fig. 3(II). Finite difference techniques were used to simulate a few broadband transient responses of the modulated metabeam Appendix C. The spectral content of these responses was obtained using discrete Fourier transform in space and in time and then used to numerically approximate the dispersion curve. Both numerical and asymptotic approximations match closely as shown on Fig. 3(II). Unless otherwise specified, illustrated results are obtained with the parameters $\rho/\rho' = 1$, $q_m = 1.25Q$, $\omega_m = 0.25\ \Omega$ and $\delta k/k = 0.1$ with $Q = \sqrt[4]{k/G}$.

The dispersion curve reveals the existence of a couple of directional bandgaps whereby waves are blocked over a frequency