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Viscous second gradient porous materials for bones reconstructed with bio-resorbable grafts

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ABSTRACT

It is well known that size effects play an important role in the mechanical behavior of bone tissues at different scales. In this paper we propose a second gradient model for accounting these effects in a visco-poro-elastic material and present some sample applications where bone is coupled with bioresorbable artificial materials of the kind used in reconstructing surgery.

1. Introduction

Substantial size effects are known to occur in the elastic behavior of (i) single osteons [1], (ii) human compact bone [2–6], (iii) human trabecular bone [7,8]. In the first case, the size effects are attributed to compliance of the interfaces between laminae. In the second case, there is experimental evidence that the cement lines as compliant interfaces account for most of the difference in stiffness between osteons and whole bone. In the third case, continuum properties vary by more than 20%–30% over a distance spanning three to five trabeculae and hence a continuum model for the structure is suspect [7]. Therefore, Ramézani et al. [8] used the Cosserat theory to describe the hierarchical multi-scale behavior of trabecular human bone using micro-CT images, namely: (i) macroscale, dealing with cancellous bone or spongy bone at real size; (ii) meso-scale, representing non-homogeneous and stochastic network clusters; (iii) micro-scale, indicating the micro-randomness and heterogeneous deformations; (iv) sub-micro- and nano-scale, showing single lamellas including collagen fibers and apatite crystals. Generally speaking, the limitations of the continuum assumption appear in two areas: near biologic interfaces, and where there are large stress gradients. To incorporate the scale of the microstructure of a heterogeneous material within the continuum framework, a number of phenomenological 'remedies' have been proposed that involve the relaxation of the local action hypothesis of classical continuum mechanics. Such enriched (or enhanced) continuum models aim at including information on the microstructure and can be classified into three main groups [9], namely: (i) non-local integral models [10,11], (ii) higher-order gradient models [12–14] and (iii) micropolar theories [15–17]. Bleustein [18] showed how the boundary conditions of a linear theory of an elastic continuum with microstructure [19] can be reduced to those of a corresponding linear form of a strain gradient theory [20]. Following this way of thinking, second gradient materials can be interpreted as a particular limit case of micromorphic (or micropolar) media because they can be deduced from micromorphic ones by constraining the micromorphic kinematic descriptors to be equal to the classical strain ones by introducing internal constraints and Lagrange multipliers. We remark that this constrained approach which is rigorous in a finite-dimensional space, it is assumed reasonably acceptable in an infinite-dimensional space on the basis of an argument of analogy. This paper is inspired by the more general framework of a research oriented to design the mechanical characteristics of the biomaterial constituting the graft, namely mass density and resorption velocity, in order to optimize the mass density distribution of the

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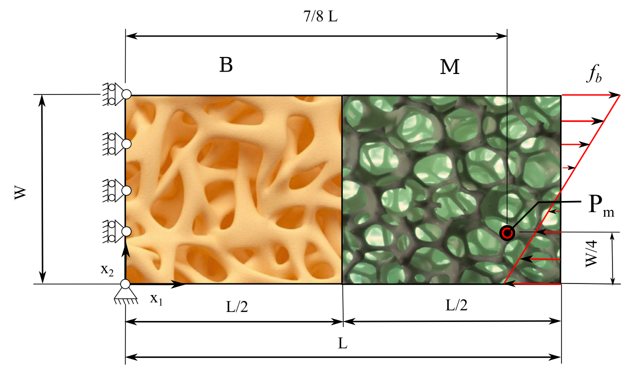

Fig. 1. Sample under study at initial stage. The labels "B" and "M" stand for bone and graft material, respectively.

growing bone tissue. The continuum model employed in this paper is accordingly richer than standard Cauchy continuum, including higher gradients of displacement in the deformation energy. The addition of terms in the energy involving second gradient of the displacement arises from the consideration of the geometry of the trabecular structure of the bone. Trabeculae are indeed organized (locally) as a lattice system oriented along the principal stress directions [21]. Since a not negligible amount of deformation energy is stored in the form of bending of trabeculae [22], a classic Cauchy model is not sufficiently rich and instead terms in the energy describing the curvature of the microstructure have to be considered. This fact naturally leads to second gradient energy models. More generally, it has been proven [23] that high contrast at micro-level of mechanical properties can impose at macro-level the need of introducing deformation energies depending on higher displacement gradient. In general, generalized continuum theories such as couple stress and micropolar have degrees of freedom in addition to those of classical elasticity [24,25]: however, in the case of the second gradient models this is not needed [18]. All such theories are thought to be applicable to materials with fibrous or granular structure. Experimentally Yang and Lakes measured the effect of size on apparent stiffness of compact bone in quasi-static torsion [3] and bending [4].

2. Material and methods

The considered specimen is constituted by the union of two bi-dimensional square portions, one constituted of bone tissue and the other of biomaterial; the square size is $L/2 = W = 0.5$ cm. The mass densities of the two materials are initially assigned in each zone and they will evolve in the subsequent remodeling process according to the mechanical and biological laws presented in the following (see Eq. (17)). The support conditions on one edge are shown in a self-explanatory way in Fig. 1.

A traction distribution corresponding to a pure bending is applied to the opposite edge as shown in Fig. 1; the load is harmonically variable with a low frequency Ω in order to activate the component of the stimulus which is related to dissipation, because this phenomenon plays a key role in the bone functional adaptation, as discussed in [26]. In particular, we set

$$f_b(x_2, t) = \left(\frac{2x_2}{W} - 1\right)\left[F_0 + F_1 \sin(\Omega t)\right]. \tag{1}$$

Some relevant results will be presented with reference to the probe point P_m in the material zone (Fig. 1).

3. Governing equations

Kinematics. In order to give a macroscopic description of the system under study constituted by an insert of bio-resorbable grafting material and a piece of bone, i.e. a porous mixture, we introduce the placement field:

$$\boldsymbol{\chi} : (\boldsymbol{X}, t) \mapsto \boldsymbol{x} \tag{2}$$

which takes each point of body $\boldsymbol{X}$ in the reference configuration $\mathcal{B}$ and time $t \in \mathbb{R}$ into a place $\boldsymbol{x}$ in the current configuration. Therefore, we consider the solid-matrix macroscopic displacement ($\boldsymbol{u} = \boldsymbol{x} - \boldsymbol{X}$) as a basic kinematical descriptor and use the Saint–Venant strain tensor

$$E_{ij}(\mathbf{X}, t) = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{i,k}\, u_{k,j}\right) \tag{3}$$

to take elastic deformations into account. Because of the porous nature of our system, we introduce another independent kinematical descriptor to describe the micro-deformations of pores inside the solid matrix of the system. In particular, we introduce the change of the Lagrangian porosity, i.e. the change of the effective volume of the fluid content per unit volume of the body with respect to an equilibrium volume [27]. In detail,

$$\zeta(\mathbf{X}, t) = \phi(\boldsymbol{\chi}(\mathbf{X}, t), t) - \phi^*(\mathbf{X}, t) \tag{4}$$

where ϕ and ϕ^* are the Lagrangian porosity related to the current and the reference configuration, respectively. By adopting the approach of the mixture theory, these porosities can be expressed as follows

$$\phi = 1 - (\rho_b/\hat{\rho}_b + \rho_m/\hat{\rho}_m), \qquad \phi^* = 1 - (\rho_b^*/\hat{\rho}_b + \rho_m^*/\hat{\rho}_m) \tag{5}$$

where ρ_b and ρ_m are the apparent mass densities of bone tissue and artificial material, respectively; the superimposed hat denotes the true densities, while the superscript * indicates all quantities in the reference configuration.

Variational equation of motion. As already mentioned, the bone is organized at micro-level as a three-dimensional porous network of interconnected trabeculae (cancellous bone). It can be also seen as a quasi-periodic system of cylindroid structures, i.e. osteons, (cortical bone) characterized by a high contrast of mechanical properties between bending and extension. Therefore, using the classical framework of Poromechanics (Biot [27], Cowin [28]) and second gradient continua (Mindlin [19], Toupin [20]), for the potential energy-density – potential energy per unit of macro-volume – we take a homogeneous, quadratic function of the variables $\boldsymbol{E}$, $\nabla\boldsymbol{E}$, ζ and $\nabla\zeta$ [29,30,13]

$$\begin{aligned}\mathscr{E} = {} & \frac{1}{2}\lambda(\rho_b^*, \rho_m^*)\, E_{ii}E_{jj} + \mu(\rho_b^*, \rho_m^*)\, E_{ij}E_{ij} \\ & + 4\,\alpha_1(\rho_b^*, \rho_m^*)E_{ii,j}E_{jk,k} + \alpha_2(\rho_b^*, \rho_m^*)E_{ii,j}E_{kk,j} \\ & + 4\,\alpha_3(\rho_b^*, \rho_m^*)E_{ij,i}E_{kj,k} + 2\,\alpha_4(\rho_b^*, \rho_m^*)E_{ij,k}E_{ij,k} \\ & + 4\,\alpha_5(\rho_b^*, \rho_m^*)E_{ij,k}E_{ik,j} + \frac{1}{2}K_1(\rho_b^*, \rho_m^*)\zeta^2 \\ & + \frac{1}{2}K_2\zeta_{,i}\zeta_{,i} - K_3(\rho_b^*, \rho_m^*)\,\zeta\, E_{ii}\end{aligned} \tag{6}$$

where λ and μ are the Lamé parameters

$$\lambda = \frac{\nu\, Y(\rho_b^*, \rho_m^*)}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{Y(\rho_b^*, \rho_m^*)}{2(1+\nu)}, \tag{7}$$

here expressed in terms of the Young modulus of the mixture

$$Y = Y_b^{Max}\left(\rho_b^*/\hat{\rho}_b\right)^2 + Y_m^{Max}\left(\rho_m^*/\hat{\rho}_m\right)^2 \tag{8}$$

and Poisson ratio. Y_b^{Max} and Y_m^{Max} are the maximal elastic moduli. The second gradient stiffness coefficients are assumed to be:

$$\alpha_1 = \alpha_2 = \alpha_4 = Y(\rho_b^*, \rho_m^*)\ell^2, \qquad \alpha_3 = 2Y(\rho_b^*, \rho_m^*)\ell^2, \\ \alpha_5 = 1/2Y(\rho_b^*, \rho_m^*)\ell^2 \tag{9}$$