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Attaining the rate-independent limit of a rate-dependent strain gradient plasticity theory

S.A. El-Naaman *, K.L. Nielsen, C.F. Niordson

Department of Mechanical Engineering, Solid Mechanics, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark

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ABSTRACT

The existence of characteristic strain rates in rate-dependent material models, corresponding to rate-independent model behavior, is studied within a back stress based rate-dependent higher order strain gradient crystal plasticity model. Such characteristic rates have recently been observed for steady-state processes, and the present study aims to demonstrate that the observations in fact unearth a more widespread phenomenon. In this work, two newly proposed back stress formulations are adopted to account for the strain gradient effects in the single slip simple shear case, and characteristic rates for a selected quantity are identified through numerical analysis. Evidently, the concept of a characteristic rate, within the rate-dependent material models, may help unlock an otherwise inaccessible parameter space.

1. Introduction

Strain gradient plasticity theories have become an established part of contemporary solid mechanics due to the increasing interest in micron and nano scale plasticity. Experiments have demonstrated that size-dependent behavior, in terms of increased hardening and/or strengthening, is associated with spatial gradients of plastic strain in ductile crystalline materials (see e.g. [1,2]). The plastic strain gradients are accommodated by a portion of lattice defects, often referred to as geometrically necessary dislocations (GNDs), which leads to a long range internal stress field. Many gradient theories of plasticity have been proposed to capture size-effects, and although so-called lower order theories have been explored (e.g. [3–5]), the common approach involves theories of a higher order nature, which enable micro-structural boundary conditions (e.g. [1,6–13]). The present study employs the rate-dependent non-work conjugate type (or back stress based) higher order theory formulated by Kuroda and Tvergaard [14,15]. In this type of theory the virtual work principle remains the conventional one, while the evolution of GND densities is accounted for through additional differential equations. Here, a back stress, representing the long range internal stresses due to pile-up of GNDs, affects the plastic slip rate as kinematic hardening.

In the following study the existence of a characteristic slip rate, at which a specific macroscopic quantity becomes independent of the rate sensitivity exponent, will be demonstrated through numerical analysis of the idealized simple shear case for a single crystal. The adopted methodology represents a promising tool for obtaining rate-independent results using rate-dependent frameworks, and the extent of the matter remains to be explored. The idea of a characteristic rate was first discussed in detail by Nielsen and Niordson [16] in relation to conventional rate-dependent steady-state modeling and later exploited in [17] to extract rate-independent results from a scale-dependent steady-state framework. Nielsen [18] also found similar results for steady-state sheet rolling. Characteristic rates may exist for a wide range of other structural problems, and a broader sense of the phenomenon is demonstrated through the results of the present study.

* Corresponding author. Tel.: +45 4525 4020; fax: +45 4593 1475.
E-mail address: saeln@mek.dtu.dk (S.A. El-Naaman).

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The paper is structured as follows. The strain gradient plasticity model and adopted back stress formulations are briefly outlined in Section 2, (details can be found in [14,15,19,20]). The boundary value problem treated is described in Section 3, after which, a series of numerical results are presented in Section 4. The study is concluded in Section 5.

2. Strain gradient crystal plasticity model

The present study employs the strain gradient crystal plasticity theory proposed by Kuroda and Tvergaard [14,15] within a conventional rate-dependent small strain elasto-viscoplastic framework. Hence, the total strain rate is given by; $\dot{\varepsilon}_{ij} = \left(\dot{u}_{i,j} + \dot{u}_{j,i}\right)/2$, which is additively decomposed into an elastic part, $\dot{\varepsilon}^e_{ij}$, and a plastic part, $\dot{\varepsilon}^p_{ij}$, so that $\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^p_{ij}$. Plastic deformation occurs as a result of crystallographic slip on the individual slip systems, and thus, the Cartesian components of the plastic strain rate is given in terms of the slip rate, $\dot{\gamma}$, on the α'th slip system, as

$$\dot{\varepsilon}^p_{ij} = \sum_\alpha \dot{\gamma}^{(\alpha)} P^{(\alpha)}_{ij}, \qquad P^{(\alpha)}_{ij} = \frac{1}{2}\left(s^{(\alpha)}_i m^{(\alpha)}_j + m^{(\alpha)}_i s^{(\alpha)}_j\right). \tag{1}$$

The superposed dots denote material time derivative, $P^{(\alpha)}_{ij}$ is the Schmid orientation tensor, and unit vectors $s^{(\alpha)}_i$ and $m^{(\alpha)}_i$ specify the slip direction and the slip plane normal, respectively (see Fig. 1). The equilibrium equations for the non-work conjugate formulation are given by conventional stress equilibrium in absence of body forces; $\sigma_{ij,j} = 0$, where the Cauchy stress rate tensor is given by; $\dot{\sigma}_{ij} = \mathscr{L}_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^p_{kl}\right)$, in which $\mathscr{L}_{ijkl}$ is the fourth order elastic stiffness tensor. Thereby, the conventional incremental principle of virtual work reads

$$\int_V \mathscr{L}_{ijkl}\dot{\varepsilon}_{kl}\delta\dot{\varepsilon}_{ij}\mathrm{d}V = \int_V \mathscr{L}_{ijkl}\dot{\varepsilon}^p_{kl}\delta\dot{\varepsilon}_{ij}\mathrm{d}V + \int_S \dot{T}_i\delta\dot{u}_i\mathrm{d}S, \qquad \dot{T}_i \equiv \dot{\sigma}_{ij}n_j, \tag{2}$$

where $\dot{T}_i$ are the traction rates, n_i is the outward unit normal to the surface S bounding the volume V. In a two dimensional setting, an additional partial differential equation accounts for the evolution of GND density; $1/b\gamma^{(\alpha)}_{,i}s^{(\alpha)}_i + \rho^{(\alpha)}_G = 0$, where b is the magnitude of the Burgers vector, and $\rho^{(\alpha)}_G$ is the GND density of edge type on slip system α [21]. The GND density balance equation is expressed on weak form as

$$\frac{1}{b}\int_V \delta\rho_{,i}s^{(\alpha)}_i\gamma^{(\alpha)}\mathrm{d}V = \frac{1}{b}\int_S \delta\rho\zeta^{(\alpha)}\mathrm{d}S + \int_V \delta\rho\rho^{(\alpha)}_G\mathrm{d}V, \qquad \zeta^{(\alpha)} \equiv \gamma^{(\alpha)}n_i s^{(\alpha)}_i, \tag{3}$$

where $\delta\rho$ is a weighting function (or virtual GND density).[1]

A perfectly plastic, gradient-enhanced version of the widely used conventional power law slip rate relation [22,23] is employed, so that

$$\dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \mathrm{sgn}\left(\tau^{(\alpha)} - \tau^{(\alpha)}_b\right)\left(\frac{|\tau^{(\alpha)} - \tau^{(\alpha)}_b|}{\tau_0}\right)^{1/m}, \tag{4}$$

where $\dot{\gamma}_0$ is a reference slip rate, $\tau^{(\alpha)}$ is the Schmid stress ($\tau^{(\alpha)} = \sigma_{ij}P^{(\alpha)}_{ij}$), $\tau^{(\alpha)}_b$ is a back stress, m is the rate sensitivity exponent, and τ_0 is the critical resolved shear stress.[2] The back stress, $\tau^{(\alpha)}_b$, is phenomenologically related to the distribution of the GND density, and accounts for the long range internal stresses due to dislocation pile-up. In the present study, two back stress formulations, proposed in [19], are adopted. One is a thermodynamically consistent formulation derived from a free energy potential

$$\tau^{(\alpha)}_b = \mu\tau_0 b^\mu L^{\mu+1}\left(\left|\rho^{(\alpha)}_G\right| + \rho_0\right)^{\mu-1}\rho^{(\alpha)}_{G,i}s^{(\alpha)}_i, \qquad 0 < \mu \leq 1, \tag{5}$$

where ρ_0 is a non-zero numerical parameter, which resembles the presence of statistically stored dislocations (see e.g. [24,9]). Note that Eq. (5) corresponds to a quadratic free energy for $\mu = 1$.

The second back stress relation employed in the present study is given by the piece-wise function,

$$\tau^{(\alpha)}_b = \begin{cases} b\tau_0 L^2 \rho^{(\alpha)}_{G,i}s^{(\alpha)}_i & , \text{for } |\tau^{(\alpha)}_b| \leq \tau_T \\ \mathrm{sgn}\left(\rho^{(\alpha)}_{G,i}s^{(\alpha)}_i\right) b^\kappa \tau_T^{1-\kappa}\tau_0^\kappa L^{2\kappa}\left|\rho^{(\alpha)}_{G,i}s^{(\alpha)}_i\right|^\kappa & , \text{for } |\tau^{(\alpha)}_b| > \tau_T \end{cases}, \tag{6}$$

where $0 \leq \kappa \leq 1$ is assumed and τ_T defines a transition point, from a quadratic free energy based back stress, into a power law dependence on the GND density gradients. Note that Eq. (6) corresponds to a quadratic free energy for $\kappa = 1$, but thermodynamical consistency is not guaranteed for other values of κ. However, the numerical solutions presented, have been found to satisfy positive dissipation throughout the loading history, such that; $\sigma_{ij}\dot{\varepsilon}^p_{ij} = \sum_\alpha \tau^{(\alpha)}\dot{\gamma}^{(\alpha)} \geq 0$.

For a detailed description of the adopted back stress formulations and choice of model parameters see [20,19].

3. Boundary value problem

The single slip simple shear problem, with positive slip in the x_2-direction, is considered in order to demonstrate the rate-dependent behavior of the adopted back stress based strain gradient plasticity model (see Fig. 1 for a schematic illustration and definition of the slip system). The following model parameters are used throughout: Young's modulus $E = 130$ GPa, Poisson's ratio $\nu = 0.3$, $\tau_0 = 50$ MPa, and $b = 0.286$ nm. The following two back stress model parameters are used: $\rho_{G,0} = 10^5$ mm^{-2} and

1 For a detailed discussion on the micro-structural boundary conditions see [15].

2 Note that for $\tau^{(\alpha)}_b$ equal to zero, Eq. (4) reduces to the conventional theory.