Extreme Mechanics Letters 9 (2016) 91–96

Contents lists available at ScienceDirect

Extreme Mechanics Letters

journal homepage: www.elsevier.com/locate/eml

Harnessing structural hierarchy to design stiff and lightweight phononic crystals

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ARTICLE INFO

Article history:
Received 4 April 2016
Received in revised form 18 May 2016
Accepted 18 May 2016
Available online 24 May 2016

Keywords:
Phononic crystals
Hierarchical
Wave propagation
Honeycombs
Band gaps

ABSTRACT

In this letter we report a class of hierarchically architected honeycombs in which structural hierarchy can be exploited to achieve prominent wave attenuation and load-carrying capabilities. The hierarchically architected honeycombs can exhibit broad and multiple phononic band gaps. The mechanisms responsible for these band gaps depend on the geometric features of the hierarchical honeycombs rather than their composition. Furthermore, the introduction of structural hierarchy also endows the hierarchical honeycombs with enhanced stiffness. We predict that the proposed hierarchical honeycombs can realize a unique combination of wave attenuation and load-carrying capabilities, thereby providing opportunities to design lightweight and stiff phononic crystals for various engineering applications.

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Phononic crystals have gained increasing research interests because of their rationally designed periodic architectures and compositions enabling to modify phononic dispersion relations, thereby providing avenues to tailor group velocities and hence the flow of vibrational energy [1]. When the structural periodicity of phononic crystals is comparable to the wavelength of propagating waves, Bragg interference of elastic waves scattered by the compositions arises. This mechanism gives rise to complete wave band gaps: frequency ranges where incident elastic waves are not allowed to propagate. This fundamental property offers a variety of promising applications, including wave filtering [2,3], waveguiding [4–6], and energy harvesting [7–9]. However, the inherent architectures and compositions, if not designed properly, could not generate desired wave band gaps and even lead to mechanical instability that is inapplicable for load-carrying conditions.

Structural hierarchy has been employed as an important strategy to explore improved mechanical properties and other unusual physical properties. Typical examples range from Eiffel tower to hierarchically architected nanotrusses with multiple length scales [10–12]. Recent studies show that it is possible to manipulate wave propagation by harnessing multiscale characteristic of hierarchical architectures [13–15]. These rationally designed hierarchical architectures can give rise to multiple and broadband phononic band gaps as well as low frequency band gaps. In addition, these hierarchical architectures also enable the coexisting of multiband wave filtering and waveguiding in an ultrawide frequency range [16].

Despite these considerable efforts, challenges still remain. For example, to achieve the desired wave attenuation, soft materials using thermally coupled dissipation mechanism are often employed in engineering practice. As a result, the wave attenuation capability strongly depends on the thickness of the materials, thus posing a great challenge to design lightweight and stiff materials with strong wave attenuation ability. Furthermore, conventional phononic crystals with periodic architectures can only provide limited frequency band gaps since the Bragg interference requires that the wavelength must be comparable to the given structural periodicity. To overcome

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http://dx.doi.org/10.1016/j.eml.2016.05.009
2352-4316/Published by Elsevier Ltd.

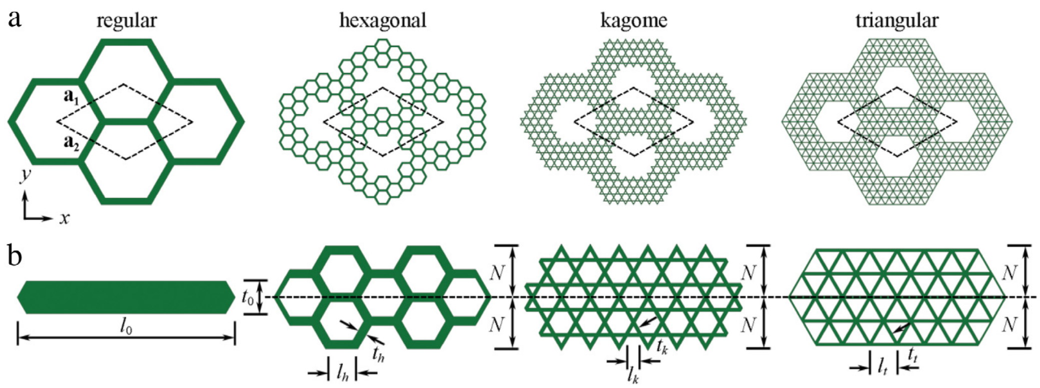

Fig. 1. (a) Schematics of regular honeycomb and hierarchical honeycombs. Here $\mathbf{a_1} = (3l_0/2, \sqrt{3}l_0/2)$, $\mathbf{a_2} = (3l_0/2, -\sqrt{3}l_0/2)$ are lattice constants; l_0 and t_0 are the length and thickness of cell walls. The dash lines indicate the supercells; (b) schematics of cell walls of regular honeycombs hexagonal, kagome, and triangular hierarchical honeycombs, respectively.

the bandwidth limitation, numerical approaches such as topology optimization have been developed to maximize the band gap size [17,18]. It is worth noting that the objective function of this approach is to maximize a single band gap size, and the resultant architectures are still spatially periodic. Lightweight and stiff phononic crystals with broadband and multiband wave attenuation ability remain unrealized.

Here, we report a class of hierarchically architected honeycombs in which structural hierarchy is exploited to simultaneously improve the wave attenuation and load-carrying capabilities. The proposed hierarchical architectures are constructed by replacing the cell walls of the regular honeycombs with hexagonal, kagome, and triangular lattices, respectively (referred to as hexagonal, kagome, and triangular hierarchical honeycombs for simplicity in the following, Fig. 1(a)–(e)). For the purpose of fair comparison, kagome and triangular hierarchical honeycombs are subsequently obtained by connecting the midpoints and vertices of the hexagonal lattice, respectively. The proposed hierarchical honeycombs are characterized by two geometric parameters, hierarchical length ratio, $\gamma = l_h/l_0$, and the number of hexagonal lattice away from the central axis, N, where l_0 and l_h are the length of cell walls of regular lattice and hexagonal lattice, respectively. The length and thickness of the hexagonal, kagome, and triangular lattices are determined by mass/volume equivalence between regular honeycombs and hierarchical honeycombs (See Supporting Information, Appendix A). The composition of the regular honeycombs and hierarchical honeycombs is a glassy polymer, SU-8, whose properties are characterized by a Young's modulus $E_s = 3.3$ GPa, Poisson's ratio $\nu = 0.33$, a yield stress $\sigma_y = 105$ MPa, and density $\rho_s = 1200$ kg/m^3 [19].

To investigate the wave attenuation capability of the proposed hierarchical honeycombs, phononic dispersion relations are constructed by performing eigenfrequency analysis within the finite element framework using the commercial package COMSOL Multiphysics. Note that we focus on the in-plane wave propagation in the hierarchical honeycombs, thus a plane strain assumption is made without loss of generality. To capture the periodic feature of the hierarchical honeycombs, Bloch's periodic boundary conditions are applied at the boundaries of the supercell. The supercell is discretized using 6-node triangular elements. We then solve the wave equation by scanning the wave vectors in the first irreducible Brillouin zone. More details concerning the modeling of wave propagation are provided in the Supporting Information (see Appendix A).

We start by examining the phononic dispersion relations of hierarchical honeycombs with $\gamma = 1/5$, $N = 1$, and relative density $\rho/\rho_s = 0.06$. For the purpose of comparison, phononic dispersion relation of the associated regular honeycomb is also reported. For the regular honeycomb, we only observe one narrow band gap at $\varpi = 0.059$–0.061 (Fig. 2(a)). By contrast, the introduction of structural hierarchy in the regular honeycombs leads to much broader band gaps (Fig. 2(b)–(d)). Specifically, the maximum band gaps in hexagonal, kagome, and triangular hierarchical honeycombs are $\varpi = 0.047$–0.079, $\varpi = 0.108$–0.133, and $\varpi = 0.064$–0.078, respectively. In addition, the introduction of structural hierarchy also gives rise to multiple band gaps, as shown in the phononic dispersion relations. To gain a deeper understanding, we plot the eigenmodes of the high-symmetry points $\bar{\Gamma}$, $\bar{M}$, and $\bar{K}$ at the lower band edges of the band gaps (Red lines in Fig. 2). For hexagonal and kagome hierarchical honeycombs, the vibrational modes of the high-symmetry points exhibit a global nature, indicating a Bragg-type band gap. Interestingly, localized vibrational modes are observed for the triangular hierarchical honeycombs, suggesting that local resonances are responsible for the broad band gaps [20–23]. This is also supported by the flat band edge of the band gaps. A direct comparison between the geometric features of the regular honeycomb and hierarchical honeycombs leads us to believe that different mechanisms of band gaps formation are intrinsically dictated by the slenderness ratio and coordination number of the lattice. It should be pointed out that damping effect resulting from the viscoelastic feature of the glassy polymer may make some contribution to the wave attenuation [24]. However, recent experimental results indicate that the damping effect will not swamp the band gaps in the phononic dispersion relations [25].