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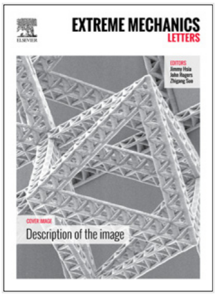

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Buckling of elastic beams embedded in granular media

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In this paper, an experimental and theoretical study of the buckling response of elastic slender beams within granular media is performed. Buckling loads of beams with different flexural rigidity, length, and boundary conditions within granular media of different depths are determined. The Ritz approximate method is implemented to model the buckling response of the beams based on the concept of an overhanging beam on elastic foundation, using a series of springs whose spring constants change linearly with respect to the depth of the grains. There is good agreement between the experimental results and the theoretical model.There is a characteristic penetration ratio where the beams are not able to sense the boundary condition at the embedded end, resulting in a convergence of the buckling loads. This condition happens when the rigidity of the beam is lower than the effective stiffness of granular support, leading to the confinement of the lower portion of the beam inside the grains, and acting as a secondary boundary condition that is independent of the condition at the end of the beam. We derive a scaling law to characterize this characteristic penetration ratio in terms of a dimensionless stiffness parameter, allowing for the characterization of three distinct interactions between the beam and medium based on the ratio of granular support effective stiffness to the beam's effective stiffness.

Keywords: buckling; beams; granular media; elastic foundation

Introduction Slender rods and beams, commonly referred to as *elastica*, can be used to describe carbon nanotubes, fiber optic cables, spider silk, and human hair. In many engineering applications, it is desirable to insert and manipulate an elastic beam within complex media, such as granular beds or soft tissues. Burrowing a flexible structure through fragile media requires understanding the coupled interactions between the geometrically nonlinear structure and its reconfigurable surroundings. The complex interplay between the elastic strip and its surroundings has been studied as the beam is lowered into a fluid [1], constrained along its edges [2], delaminated from a surface [3], embedded on an elastomeric matrix [4, 5], and compressed while resting on an elastic foundation [6–10] and floating on the surface of water [11]. The fundamental structural interactions of the constrained elastica have direct analogies to plant root growth [12–14], bending and buckling of oil pipelines [15, 16], and the creation of underground infrastructures using microtunneling and trenchless technology [17].

The interaction between a flexible beam and a granular medium can be modeled as a beam on an elastic foundation with a series of springs acting along the length of the beam [18]. Winkler's model has been widely used in problems involving soil–structure interactions when pile–supported structures *e.g.* bridges and piers, transfer load to the surrounding foundation *via* lateral, shear, and/or axial interaction. The buckling behavior of the pile was modeled by assuming that the soil subgrade modulus, which is the ratio of lateral soil reaction per unit length over its lateral deflection, is constant and linearly increasing with depth [19]. This analysis was later extended to a general power distribution of the soil subgrade modulus with respect to the depth to find the critical buckling load of partially embedded piles in soil [20]. The energy method was used to obtain the buckling load of the piles embedded in soil with different boundary conditions [21–23]. The effect of skin friction, the applied friction force per unit length from the soil to the sides of the pile, on the buckling load of embedded piles was found to be insignificant, resulting to less than 10% variation in the buckling load [20, 24, 25]. This negligible friction effect could be the result of a small friction force along the length of the pile compared to the applied critical buckling load, and also small displacement in the vertical

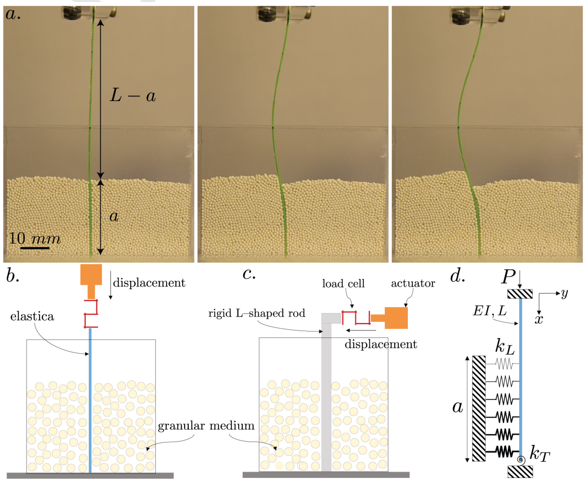

FIG. 1: *a*. An elastic beam partially embedded within a granular medium. *b*. The experimental setup for the buckling experiment *c*. The experimental setup for the lateral force-displacement experiment *d*. Winkler model used in the theoretical model