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# Enhanced elastic-foundation analysis of balanced single lap adhesive joints

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ABSTRACT

Conventional single-lap adhesive joints between identical adherends achieve ultimate strength only after significant inelastic deformation of the adhesive and perhaps also the adherends. However purely elastic analysis provides insights and is relevant to fatigue initiation or brittle failure. We extend classical beam-based elastic results, both 'within the bond' (deriving more-accurate peak peel stress from the joint-edge moment) and 'beyond the bond' (determining the edge moment from adherend dimensions, remote boundary conditions, and load).

Within the bond, we show that peak adhesive equivalent stress and principal stress are minimized when the bond length exceeds four characteristic lengths of the elastic-foundation shear stress equation. This makes simplified 'long' joint formulas attractive for initial design. We then examine how well the long-joint predicted peak peel stress matches plane strain finite element analysis, and empirically capture a peel-stress end effect due to nonzero adhesive Poisson ratio. With this end-effect correction, the limit of useful accuracy can be expressed as a ratio $R_a$ of (adherend *axial* stiffness) to (adhesive *axial* stiffness) being $\gtrsim$ a number of order $10^2$-$10^3$ depending on Poisson ratio. This limit supplements the Goland and Reissner proposed applicability limit for elastic foundation analysis, expressed as a limiting ratio $R_v$ of *through-thickness* or vertical stiffnesses.

Outside the bond, Timoshenko-style beam-column expressions are used to derive a simplified joint-edge moment factor. While similar in spirit to the edge-moment determination of Goland and Reissner for infinite-length pinned adherends, treating the bond region as a rigid block leads to simpler nonlinear expressions, and captures the moment-reducing benefits of shorter (finite-length) adherends and fixed-slope end conditions. Joint rotation effects become dominant when $TL^2 > \overline{E}I$ ($L$ is adherend free length, $T$ is tensile load), then joint rotation magnitude depends on $TD^2/\overline{E}I$ ($D$ is lap length).

## 1. Introduction

The single lap joint (SLJ) is common because fabrication is so convenient. The need for adherend dimensional precision is low, and virtually no forming or machining is required. The already-flat surfaces of the bars or sheets to be joined are simply overlapped with adhesive, squeezed together, and fixtured at a desired separation until the adhesive hardens [1]. While early analyses of lap joint adhesive stress were strictly elastic in character, it is now recognized that the ultimate strength of aerospace sheet metal bonds is developed only after plastic straining of the adhesive and possibly adherends. Even so, simple elastic analyses are not irrelevant as they provide a foundation for understanding joint mechanics. In addition, there may be joints for which brittle fracture or fatigue initiation are a greater concern than ultimate strength involving plasticity. It is from that perspective that this purely elastic investigation was conducted.

Goland and Reissner [2] introduced the partition of lap joint analysis into 'inner' and 'outer' problems. For the inner problem they assumed the application of joint-edge force and bending moment. Then for the case of significant through-thickness adhesive compliance, they developed the well-known approximate beam-on-elastic-foundation model, and computed peak peel stress due to those edge loads. For the outer problem, they used the governing equation for a beam with tension (the adherend), connected to a finite-length double-thickness beam (the joint region). Their main resulting formula gave the edge moment applied to the joint region, as a function of load, for the case of infinite length adherends with moment-free end supports. Of course, the foregoing is far from a complete list of the accomplishments in their seminal paper.

Although credible numerical elastic-plastic nonlinear analyses are now routine, specific quantitative results are not an ideal design tool. One also needs insight into trends and limits, and if possible, simple algebraic estimates to guide a design approach. The purpose of this investigation is to extend certain aspects of elastic lap-joint analysis,

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with the desired outcome of useful simple formulas.

In Section 2, we justify a focus on 'long' joints which exhibit length-independent peak stresses. We begin with well-known elastic-foundation formulas for averaged peel and shear stress in the adhesive of finite-length joints (such through-thickness smeared values are quite representative of adhesive midline stresses, as long as the joint is well modeled as beams connected by an elastic foundation – according to Goland and Reissner [2], that is when the elastic layer vertical compliance is not too small). Assuming approximately zero midline axial strain, plane-strain elastic relations are used to approximate all adhesive stress components from the peel and shear stress, permitting the equivalent stress and greatest principal stress to be computed. Since their peak values are always found at the joint ends (apart from a numerically determined end-effect stress reduction covered in Section 3), we examine those joint-end values as a function of overlap length. It is observed that the peak 'equivalent stress' (for both pressure-independent and pressure-sensitive yield) and peak principal stress reduce towards asymptotically minimum values as the joint is lengthened beyond about four characteristic lengths of the shear stress equation. For design purposes, it therefore seems reasonable to specify that joints should routinely exceed this minimum length. This permits use of the substantially simpler long-joint peak-stress formulas presented by Bigwood and Crocombe [3].

For such 'long' joints, Section 3 compares the peel stress $\sigma$ determined by elastic foundation analysis to $\sigma_{yy}$ on the adhesive midline computed by plane strain finite element analysis. For zero Poisson ratio of the adhesive, these match quite well all along the midline, over a large range of joint parameters. But for nonzero adhesive Poisson ratio an end effect is observed (over an axial distance proportional to the geometric mean of adhesive thickness and adherend thickness) that truncates the peel stress peak due to loss of horizontal constraint. By curve fitting we determine an empirical expression for the end-effect distance, and combine this with the beam on elastic foundation (BEF) stress solution to approximate the finite element analysis (FEA) peel stress peak (which is always lower than the unmodified BEF peak).

When the corrected elastic foundation peak peel stress is compared to the peak peel stress computed by FEA, agreement is good within a 'region of applicability' in joint-parameter space, whose boundary is based on $R_a$, the ratio of adherend to adhesive *axial* stiffness's. This supplements the well-known Goland and Reissner [2] applicability boundary for elastic foundation analysis, which may be expressed in terms of the ratio $R_v$ of *through-thickness* (vertical) stiffness's. Both criteria agree in excluding too-stiff adhesive from elastic-foundation analysis.

In Section 4 we turn to the 'outer' problem, in order to extend the Goland and Reissner [2] analysis of edge moment factor, $k$. We adapt the well-known beam-column formalism presented in Timoshenko and Gere [4], and approximate the thick overlap region as a rigid block. This allows us to give results for adherends of finite length, and include not only moment-free but fixed-slope end conditions. The resulting edge-moment expressions are both more general and simpler.

There exists an extensive literature on the simplified elastic analysis of single lap joints, as outlined in da Silva et al. [5] and extended to dynamic loading by Vaziri et al. [6,7] and others. Many investigators including Goland and Reissner [2], Volkersen [8] and Hart-Smith [9] used the elastic-foundation approach to investigate approximate through-thickness shear and peel stress distributions of a relatively compliant adhesive layer. In addition, geometrical nonlinearity, which arises from tension rotating the joint region to bring remote adherends closer to coaxial alignment, was long ago recognized by Goland and Reissner [2]. They used axially-loaded beam analysis to determine the edge moment (i.e., the adherend centroidal bending moment at the joint edge) for infinitely long, pinned adherends. Luo and Tong [10] reviewed this and other treatments of rotation.

In addition to many analytical investigations, finite element analyses in 2$D$ and 3$D$ have been performed by Adams and Peppiatt [11], Her [12], Li and Lee-Sullivan [13], Tsai and Morton [14], Goncalves *et al.* [15], Ashrafi *et al.* [16], Haghpanah *et al.* [17] among others. Some of these were elastic-only, while others included adhesive and/or adherend plasticity. To navigate this large literature, we have relied on authoritative and comprehensive reviews by Minford [18], Da Silva *et al.* [5], and Adams *et al.* [19]. In the publications we have explored, we have not encountered the results developed here.

## 2. Adhesive failure stress from elastic foundation analysis

The purpose of this section is to show that 'long' joints (defined relative to the characteristic length $\lambda_s$ of the elastic-foundation equation for adhesive shear stress) exhibit the lowest peak 'equivalent stress' responsible for yield, and also the lowest peak principal stress (responsible for brittle fracture). Assuming that designers will generally exploit this strength advantage, it seems reasonable to initiate designs with the simple 'long joint' formulas for peak stress, as provided by Bigwood and Crocombe [3].

Consider a joint with $180^o$ symmetry loaded by a force in the joint plane (thus giving rise to maximum adherend bending moment with no shear force, see Fig. 1). The well-known elastic-foundation governing equation for peel stress σ is [5]:

$$\frac{d^4\sigma}{dx^4} + \frac{4\sigma}{\lambda_p^4} = 0 \tag{1}$$

where the peel characteristic length $\lambda_p$ is defined by $\lambda_p = \sqrt{\frac{\bar{E} t^3 t_a}{6\bar{E}_a}}$.

Here we have used *equivalent Young's moduli* with an overbar, defined as follows: For the adherend it is the *plane-strain Young's modulus* $\bar{E} = E/(1-\nu^2)$, although this won't properly represent axial stretching unless $w >> max(L, D)$, where $L$ is the free adherend length, $D$ is the joint overlap and $w$ is the specimen width perpendicular to the axis of loading. For adhesive away from the edges and corners of the joint, we use the transversely constrained modulus:

$$\bar{E}_a = \frac{E_a(1-\nu_a)}{(1+\nu_a)(1-2\nu_a)} \tag{2}$$

(While this expression suggests unbounded stiffness as $\nu_a \to 0.5$, in fact $E_a$ should not be taken as independent of $\nu_a$. Polymer bulk modulus $K_a$ arising from interatomic repulsive forces is relatively unvarying in the range of 1 – 5 GPa, whereas the moduli capturing distortional behavior such as $E_a = 3K_a(1-2\nu_a)$ and $G_a = (3K_a/2)(1-2\nu_a)/(1+\nu_a)$ both reduce toward zero as Poisson ratio approaches 0.5. It can be useful to recast some of the below expressions with the adhesive moduli expressed in terms of $K_a$.).

The solution of the peel stress equation for an arbitrary length bond subjected to an applied joint-edge moment per unit width $Ft/2$ (where $F$ is force per unit width of specimen) is a well-known symmetric expression [5]:

$$\sigma = \frac{Ft}{\lambda_p^2}\left[A \sinh\left(\frac{x}{\lambda_p}\right)\sin\left(\frac{x}{\lambda_p}\right) + B \cosh\left(\frac{x}{\lambda_p}\right)\cos\left(\frac{x}{\lambda_p}\right)\right] \tag{3}$$

where

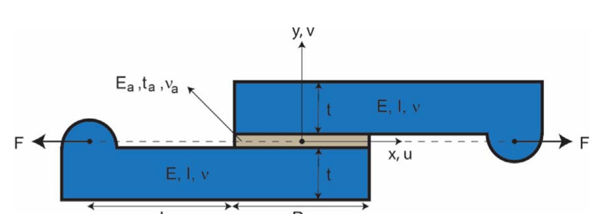


**Fig. 1.** Canonical loading of a SLJ: symmetric with no force obliquity, hence resulting in maximum edge moment.