



# Modification and evaluation of a FRF-based model updating method for identification of viscoelastic constitutive models for a nonlinear polyurethane adhesive in a bonded joint

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## ARTICLE INFO

### Keywords:

Polyurethane (A)  
Elastic modulus (D)  
Mechanical properties of adhesives (D)  
Viscoelasticity (D)

## ABSTRACT

In this study, a Frequency Response Function (FRF) -based model updating method, was modified for the purpose of the identification of viscoelastic constitutive models. A steel beam, bonded to a heavy rigid steel block by a layer of Sikaflex-252 polyurethane adhesive, was employed as the test setup. Using the concept of Optimum Equivalent Linear FRF (OELF), acceleration FRFs were measured at different random excitation levels which demonstrated the nonlinear behavior of the adhesive. Using a finite element model, the sensitivity analysis showed that the selected FRFs are more sensitive to the storage and loss moduli of the adhesive near the resonances. Therefore, firstly, both of the storage and loss moduli were identified near each resonance separately and the results have been compared with the results based on Inverse Eigen-sensitivity Method (IEM). In continuation, five viscoelastic constitutive models were utilized and identified to characterize the dynamic mechanical properties of the adhesive at different excitation levels. Applying the identified models, the correlation between the FRFs of the FE models and the experimental ones were tested. The results show that amongst the identified models, The Standard Linear Solid (SLS) model in parallel with a viscous or constant structural damper (stiffness proportional) results in better correlation with experiments. Increasing the excitation level, the storage modulus of the adhesive decreases, whereas the loss modulus increases, especially at high frequencies.

## 1. Introduction

Nowadays, there are continually growing trends toward application of polyurethane adhesives in many different industries such as wind turbines, construction, automotive and transportation. This type of adhesives requires fewer curing steps than epoxies, resulting in reduced production costs. Some of the other advantages are fatigue resistance, crack retardation and good damping characteristics. So, establishing new techniques to build and tune the Finite Element (FE) models for simulation of the static and dynamic behavior of structures with adhesively bonded joints is an increasing need. He [1] reviewed some of the published work until 2010, relating to the FE analysis of the adhesively bonded joints, in terms of static loading analysis, environmental behaviors, fatigue loading analysis and dynamic characteristics of the adhesively bonded joints.

In response to dynamic loading, most of adhesives demonstrate viscoelastic behavior that may depend on temperature, excitation frequency, excitation amplitude, pre-stress and humidity. Therefore, definition of the viscoelastic constitutive model has a crucial effect on the accuracy of the FE model of an adhesively bonded joint. The viscoelastic constitutive models (viscoelastic characteristics) are not readily available through manufacturers' data sheets in which usually static, linear characteristics of the adhesives are provided. Consequently, identification of viscoelastic constitutive models for the adhesives is an inspiring research topic in dynamic FE modelling of adhesively bonded joints.

There are extensive studies, with different methods, on the identification of viscoelastic constitutive models of the adhesives. Recently, Najib and Nobari [2] classified these methods into two categories, namely, direct and inverse identification methods, that is repeated here

*Abbreviations:* ACC, Amplitude Correlation Coefficient; EMA, Experimental Modal Analysis; FE, Finite Element; FRAC, Frequency Response Assurance Criterion; FRF, Frequency Response Function; IEM, Inverse Eigen-sensitivity Method; OELF, Optimum Equivalent Linear FRF; RFM, Response Function Method; SCC, Shape Correlation Coefficient; SLS, Standard Linear Solid model

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briefly with some instances.

In the direct methods, for a prepared test specimen, dynamic test data are obtained using a selected experimental procedure, for instances, dynamic mechanical thermal analysis (DMTA) (Barruetabena et al. [3]) or resonance testing (Maheri and Adams [4]). These dynamic data can be converted directly to the dynamic stress-strain data or equivalently to the dynamic modulus at a specific frequency or strain rate. Over a frequency range, the parameters of a viscoelastic model can be obtained by means of curve fitting.

In the category of inverse methods the main point is that the measured dynamic data cannot be converted directly to the dynamic stress-strain data in the adhesive region of the specimen. So, an inverse problem solving is preferable, even inevitable. The methods based on the FE model updating are examples of inverse methods [2,5–7].

A recent instance of the methods based on the FE model updating is the work of Najib and Nobari [2] in which they modified a model updating method based on Frequency Response Function (FRF), referred to as the Response Function Method (RFM), for identification of the parameters of the viscoelastic constitutive model. For a steel beam bonded to a heavy rigid steel block by a layer of adhesive, they measured the acceleration FRFs at different excitation levels, using the concept of Optimum Equivalent Linear FRF (OELF) and identified the parameters of the nonlinear viscoelastic constitutive model. They validated the identified nonlinear viscoelastic model through correlation tests between the FRFs of the updated FE model and the experimental ones.

In this paper, the method developed in [2] will be implemented on 5 different viscoelastic models, in order to identify their parameters and to see which one of the models gives the best prediction of the behavior of the adhesive in question. In this respect, for a beam bonded to a rigid support via a layer of elastic adhesive, the acceleration FRFs were measured experimentally, using the concept of Optimum Equivalent Linear FRF (OELF). These FRFs were used to update the FE model of the bonded beam. The results will be compared with the ones obtained based on IEM. Also, the nonlinearity effects, attributable to the excitation level, will be examined.

## 2. Formulation of RFM

For the first time, the RFM was proposed by Lin and Ewins [8]. The reader is referred to Najib and Nobari [2] for a brief introduction and to Imregun et al. [9] and Visser [10] for more details and computational aspects. The updating equation that was used in this study is [2]:

$$-\omega^2 H_{Aij}(\omega) - H'_{Xij}(\omega) = \mathbf{H}_{Ai}^T(\omega) \Delta \mathbf{Z}(\omega) \mathbf{H}'_{Xj}(\omega) \quad (1)$$

where  $\omega$  is the circular frequency in (rad/sec),  $H_{Aij}$  is the element in  $i$ -th row and  $j$ -th column of the analytical receptance matrix ( $\mathbf{H}_A$ ),  $H'_{Xij}$  is the element in  $i$ -th row and  $j$ -th column of the experimental inertance matrix ( $\mathbf{H}'_X$ ),  $\mathbf{H}_{Ai}^T$  is the transpose of  $i$ -th column of the analytical receptance matrix and  $\mathbf{H}'_{Xj}(\omega)$  is the  $j$ -th column of the experimental inertance matrix.  $\Delta \mathbf{Z}$  is the dynamic stiffness error matrix,  $\Delta \mathbf{Z} = \mathbf{Z}_X - \mathbf{Z}_A$ , where  $\mathbf{Z}_A$  and  $\mathbf{Z}_X$  are the dynamic stiffness matrices of the analytical and experimental models of structure, respectively. In practice, it is impossible to measure complete set of  $\mathbf{H}'_{Xj}$  in (1), so the unmeasured FRFs in  $\mathbf{H}'_{Xj}$  will be filled with their analytically-derived counterparts [9]. Since this is an approximation, the method will be an iterative scheme and the convergence must be checked at each iteration step [9].

## 3. Modification of the RFM for viscoelastic material properties identification

This procedure was presented in [2] and here is repeated. In the frequency domain, the Fourier transforms of stress and strain ( $\bar{\sigma}$  and  $\bar{\epsilon}$ ) are related by:

$$E^*(\omega) = \frac{\bar{\sigma}(\omega)}{\bar{\epsilon}(\omega)} = E'(\omega) + jE''(\omega) \quad (2)$$

where  $j = \sqrt{-1}$  and  $E^*$ ,  $E'$  and  $E''$  are referred to as dynamic (or complex) modulus, storage modulus and loss modulus, respectively.

The FE model, that contains two different materials, namely, adherend and adhesive, is considered to modify RFM (Eq. (1)) for identification of unknown viscoelastic properties of the adhesive ( $E'$  and  $E''$ ). The material properties of the adherend are known, whereas those for adhesive are unknown except its density. So, the dynamic stiffness matrix of the FE model can be written down as:

$$\mathbf{Z}(\omega) = -\omega^2 \mathbf{M} + \mathbf{K}_{\text{adherend}} + j\omega \mathbf{C}_{\text{adherend}} + j\mathbf{D}_{\text{adherend}} + \mathbf{K}^*(E^*(\omega))_{\text{adhesive}} \quad (3)$$

where  $\mathbf{M}$  is the complete mass matrix and  $\mathbf{K}$ ,  $\mathbf{C}$  and  $\mathbf{D}$  with the subscript "adherend" are those parts of global stiffness, viscous damping and structural damping matrix that are related to the elements of adherend portion of the model and are known.  $\mathbf{K}^*$  is the complex stiffness matrix of the adhesive part of the model, i.e.:

$$\mathbf{K}^*(E^*(\omega))_{\text{adhesive}} = \mathbf{K}'(E'(\omega)) + j\mathbf{K}''(E''(\omega)) \quad (4)$$

$\mathbf{K}'$  and  $\mathbf{K}''$  are the only parts of  $\mathbf{Z}$  that are related to the  $E'$  and  $E''$ . So, at a fixed  $\omega$ , one can write:

$$\Delta \mathbf{Z} = \Delta \mathbf{K}' + j\Delta \mathbf{K}'' \quad (5)$$

For the solid element used in this study, the element stiffness matrix is a linear function of Young's modulus. So, assuming uniform Young's modulus for the adhesive layer, Eq. (5) becomes:

$$\Delta \mathbf{Z} = \frac{\partial \mathbf{K}'}{\partial E'} \Delta E' + j \frac{\partial \mathbf{K}''}{\partial E''} \Delta E'' \quad (6)$$

Defining  $\mathbf{S}$  as the sensitivity of stiffness matrix,

$$\frac{\partial \mathbf{K}'}{\partial E'} = \frac{\partial \mathbf{K}''}{\partial E''} = \mathbf{S} = \text{constant} \quad (7)$$

Eq. (6) becomes,

$$\Delta \mathbf{Z} = (\Delta E' + j\Delta E'')\mathbf{S} = \Delta E^* \mathbf{S} \quad (8)$$

and by the definition of

$$B_{ij}(\omega) = \frac{-\omega^2 H_{Aij}(\omega) - H'_{Xij}(\omega)}{\mathbf{H}_{Ai}^T(\omega) \mathbf{S} \mathbf{H}'_{Xj}(\omega)} \quad (9)$$

Eq. (1) reduces to

$$\Delta E^*(\omega) = B_{ij}(\omega) \quad (10)$$

The Eq. (10) is the modified version of RFM (Eq. (1)) for identification of viscoelastic properties and it is a complex equation, so:

$$\begin{bmatrix} \Delta E'(\omega) \\ \Delta E''(\omega) \end{bmatrix} = \begin{bmatrix} \text{Re}(B_{ij}(\omega)) \\ \text{Im}(B_{ij}(\omega)) \end{bmatrix} \quad (11)$$

The Eq. (11) can be used to update  $E'$  and  $E''$  at each frequency point. Also, one can write down these equations for a range of frequency points and use least square solution to identify constant values of the  $E'$  and  $E''$  over a frequency range. This will be discussed more in the Section 5.

## 4. Formulation of IEM

After the original work of Fox and Kapoor [11], the formulation of the IEM has been presented by many researchers for instance by Naraghi and Nobari [7] and here is repeated in brief to be compatible with the present paper notation. Using the same notation as in Section 3, the complex eigen-value problem for the FE model can be written as:

$$(\mathbf{K}_{\text{adherend}} + \mathbf{K}'_{\text{adhesive}} + j\mathbf{K}''_{\text{adhesive}} - \lambda_r \mathbf{M}) \{\phi\}_r = 0 \quad (12)$$

where  $\lambda_r$  and  $\{\phi\}_r$  are the  $r$ -th eigen-value and eigen-vector of the model and  $\{\phi\}_r$  is normalized such that

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