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Multiaxial variable amplitude fatigue life analysis using the critical plane approach, Part II: Notched specimen experiments and life estimations

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ABSTRACT

While part I of this paper was focused on evaluating multiaxial variable amplitude fatigue life estimations for un-notched specimens, part II extends the same critical plane-based analysis procedures to situations involving notched specimens. In addition to the factors considered in the un-notched analyses, local stress concentrations, stress gradient effects, and changes in local stress state must also be accounted for in the presence of a notch. This was accomplished in the current study by coupling the Theory of Critical Distances point method with a pseudo stress-based plasticity modeling technique. Then, a modified version of the Fatemi-Socie parameter was used to calculate fatigue damage, and changes in life estimation accuracy were studied with respect to the consideration of transient material deformation behavior, crack initiation definition, and damage summation rule. Results from the notched specimen analyses were also compared to those for un-notched specimens, and some discussion is provided. While the effect of transient deformation behavior and crack initiation definition were found to be relatively small for the loading histories used in this study, changing the critical damage sum at failure had a much greater impact on life estimations. Although some of the analysis procedures investigated were able to estimate nearly all fatigue lives within a factor of 3 of experimental results, several areas were identified where there is potential for even further improvements to be made. These include issues related to the accuracy of life estimation curves, damage calculation models, and/or the modeling of material deformation behavior.

1. Introduction

In the first part of this two part publication, critical plane-based multiaxial variable amplitude fatigue life estimations were evaluated against experimental data generated using un-notched test specimens. This second part of the paper extends the same life estimation techniques to situations involving notched specimens. In addition to the aspects considered in the un-notched specimen analyses, the local stress concentration effect of a notch adds even further complexity to the analysis procedure. Depending on the notch geometry, both the magnitude and location of maximum local stresses can change with a change in the nominal stress ratio. For example, the shift in location and magnitude of maximum stresses for the specimen geometry used in this study was previously shown in [1]. This makes fatigue life estimation for non-proportional nominal loading histories particularly complex. Under these conditions, a local analysis approach must be applied, and multiple potential failure locations may need to be evaluated, in order to properly account for the variation of stresses and strains at the notch root. Changes in local stress state due to the presence of the notch may also need to be considered.

If the magnitude of the applied loading is large enough, localized plastic deformation should also be accounted for in a notch analysis. Although non-linear finite element analysis (FEA) techniques are capable of producing highly accurate local stress-strain solutions under arbitrary loading conditions, the process is computationally expensive and highly impractical in cases involving complex component geometries and/or long loading histories. Therefore, notch stress-strain estimation models are often employed, in combination with material constitutive relations and theoretical elastic (pseudo) stress-strain histories, to calculate stresses and strains at only the critical locations within a component. However, there are several different models available, and the accuracy of each can be dependent on both material and notch geometry. For example, Neuber's rule [2] has been shown to work well for blunt notches under plane stress conditions, while the

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Equivalent Strain Energy Density (ESED) approach [3] is more suitable under plane strain conditions [4]. Although modified versions of these models have been proposed, which attempt to unify their theoretical differences [5], notch rules which have been extended to correct for multiaxial plasticity based on an equivalent stress range cannot be directly applied to non-proportionally varying stress histories without the use of plasticity modeling techniques [6].

To overcome this limitation, various methods have been proposed by researchers such as Barkey et al. [7] and Köttgen et al. [8], under the framework of incremental cyclic plasticity modeling. These approaches are able to estimate notch root stresses and strains under general, proportional and non-proportional, multiaxial loading. Köttgen et al. [8] proposed pseudo stress and pseudo strain-based approaches for notch analysis which can be applied to any notch geometry and loading path through the introduction of a structural yield surface concept. The first step in the pseudo stress approach is to derive a pseudo stress-local notch strain curve. This curve represents the stress-strain response of the structure and can be derived through nonlinear FEA, direct measurement, or the application of a uniaxial approximation formula such as Neuber's rule or ESED. Next, this curve, along with the theoretical elastic (pseudo) stress history at the desired analysis location, are input into any cyclic plasticity model to calculate elastic-plastic notch strains. Finally, the elastic-plastic strains are input back into the plasticity model, along with the material cyclic stress-strain curve, to compute the corresponding elastic-plastic notch stresses. The pseudo strain approach follows a similar procedure, but with pseudo strains first being used to calculate local elastic-plastic stresses, from which local elastic-plastic strains are derived. After comparing estimations from both approaches to FEA results for a variety of notch geometries, materials, and loading paths, Köttgen et al. concluded that the pseudo strain approach is well suited for application to sharper notches, while the pseudo stress approach may be more appropriate for cases involving components with mild notches at higher load levels.

Lee et al. [9] proposed an almost identical approach to that of Köttgen et al. [8] but used a two-surface kinematic hardening plasticity model instead of a multi-surface model. Later, Gu and Lee [10] extended the same procedure by using an endochronic plasticity model to calculate local stresses and strains. For more information on notch stress-strain estimation techniques, including an evaluation of pseudo stress-based plasticity corrections for the notched specimens tested in this study, the reader is referred to [11].

Another problem often encountered in a notched specimen fatigue life analysis is the consideration of stress and/or strain gradient effects. Normally, when stress concentration factors are defined for a particular notch geometry, the values are based on the point at the notch root experiencing the largest concentration of stress or strain. However, there usually exists a steep stress/strain gradient moving away from the notch root as well. This gradient is an important consideration in a fatigue life analysis because the mechanisms which cause fatigue damage take place within a finite volume of material, rather than at a single point. Therefore, the stress-strain values used to compute damage should take into consideration the stress/strain variation over this volume. Not accounting for gradient effects in notched components often results in overly conservative fatigue life estimations [12,13].

One of the simplest and most common ways to correct for gradient effects in a fatigue life analysis is by using a fatigue notch factor, K_f, when computing local stresses and strains from a notch rule such as Neuber's or ESED. The fatigue notch factor is defined experimentally as the ratio of un-notched to notched fatigue strength at a particular fatigue life and varies depending on material properties, applied loading conditions, and notch geometry [14]. Although straightforward to implement for simpler loading conditions and component geometries, the fatigue notch factor approach has a number of drawbacks. For example, equations proposed by Neuber [15] and Peterson [16] to estimate K_f require that stress concentration factor, K_t, be defined in terms of nominal and local stresses. Additionally, both equations depend on notch root radius and an empirically derived material characteristic length. For components where either the nominal stress is not clearly defined, the notch root radius tends towards zero, or for materials where the characteristic length constant is not available, the application of these equations becomes challenging. Furthermore, for multiaxial non-proportional loadings, where the combined stress concentration effect from each applied load continuously varies throughout a cycle, the definition of fatigue notch factor can become vague.

A more recent approach to stress/strain gradient consideration, which expands on the concepts proposed by Neuber and Peterson, but overcomes the problems associated with the fatigue notch factor, is the Theory of Critical Distances [17,18]. The Theory of Critical Distances (TCD) refers to a group of several methods, based on fracture mechanics concepts, which can be used to compute averaged stress/strain values at a notch. All of the TCD methods are based on a material dependent characteristic length (critical distance), L, which is closely related to the crack transition length found from a Kitagawa–Takahashi diagram [19]. This represents the length at which fatigue damage/failure switches from being controlled by the fatigue limit to being controlled by the threshold stress intensity factor (SIF) for a given material. The following equation gives the formulation of L, where ΔK_{th} is the Mode I threshold SIF range at a given load ratio and $\Delta\sigma_o$ is the fatigue limit stress range at the same load ratio:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_o}\right)^2 \tag{1}$$

The two most common TCD approaches are the point method and the line method, although area-based and volume-based methods have also been proposed. The point method accounts for gradient effects by considering stress/strain values at a distance of $L/2$ away from the maximum stress or strain location at the notch root. The line method, on the other hand, averages the stress and/or strain variation along a line, of length $2L$, starting at the maximum stress location and moving away from the notch in a direction normal to its curvature. Although the different TCD approaches interpret the meaning of the critical distance differently when it comes to how stresses or strains are averaged, they have all been shown to generally yield similar fatigue strength values [17,18]. Additionally, TCD approaches have been shown to work well when applied to situations involving multiaxial and/or variable amplitude loading conditions [17,20]. More information on the consideration of stress gradient effects under multiaxial loading, including an evaluation of fatigue notch factor and TCD-based analysis approaches for the notched specimens tested in this study, can be found in [11].

In part II of this two part publication, consideration of the aforementioned notch effects is incorporated into the critical plane-based multiaxial variable amplitude fatigue life analysis procedures presented in part I for un-notched specimens. Resulting life estimations are then compared to experimental results generated for notched test specimens. Comparisons are also made between both the un-notched and notched specimen fatigue life estimations, and discussion is provided on the overall results of the critical plane-based analyses. While these analyses are focused on only crack initiation aspects of fatigue life, crack growth was also analyzed for the various tests performed. The results of these crack growth analyses were published previously and can be found in [21–23].