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Consideration of the runouts and their subsequent retests into *S-N* curves modelling based on a three-parameter Weibull distribution

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ABSTRACT

Considering the influence of runouts and their subsequent retests helps to determine the *S-N* curves which are used by engineers to estimate design stress levels with a high realiability. The Weibull model proposed by Castillo et al. is a unified statistical methodology to describe this phenomen. In this article, a procedure to model the *S-N* curves, subject to the constraints mentioned above is presented. Finally, two applications considering the experimental data of girders from an antique steel bridge built in 1895 and the experimental data of welded specimens made of steel S690QL are presented.

1. Introduction

The failure of a structure caused by the effects of a cyclic load much lower than its static strength was recognized by the german engineer August Wöhler[1] in 1870 [1]. The failure mentioned above starts with the occurrence and growth of micro-cracks which are the basis for macro cracks leading to final collapse.

This phenomen called fatigue can be represented through the *S-N* curves[2], and in order to model these curves several methodologies have been proposed in [2–12], some of them are shown in Table 1.

Even today, the model proposed by Basquin in 1910 [13] is still applied on official standards for steel structures such as the Eurocode 3, ISO 12107 and IIW [14,15,4,7,16]. However, these methods are not able to consider the information given by the runouts[3] which are the most expensive tests because of their long testing time and do not include a suitable statistical distribution function. Ignoring the runouts observation affects considerably the analysis of the data, and the absence of a statistical distribution does not permit predicting with a probability p the fatigue life of a structure under a significant lower stress value near to the fatigue limit [17–19]. As a matter of fact, these models only represent an elementary geometric approach which offers a limited judgement of the experimental results.

Some other authors have proposed models which include a statistical distribution and consider the runouts influence [20–22]. Nevertheless, these models include an arbitrary logarithmic linear relationship between the range of stress $\Delta\sigma$ and the number of cycles N. Additionally, for simplicity in the calculations, the proposed models in [20,21] consider a log normal distribution and the model in [22] considers a two-parameter Weibull distribution. Actually, the authors of [20–22] suggest performing an additional research considering the three-parameter Weibull distribution $W(a,b,c)$ as it is performed in this article.

Based on three-parameter Weibull distribution $W(a,b,c)$, Castillo et al. [23] proposed a probabilistic methodology in order to estimate the number of load cycles leading to failure of structural details. Unlike the traditional methods, this proposal emphasizes the stochastic nature of fatigue by considering both, the stress range $\Delta\sigma$ and the lifetime given by the number of load cycles N as random variables [24]. This fact ensures a dimensional consistency [25] and considers the influence from runouts obtained from the experiments.

The methodology mentioned above demands the estimation of two geometrical parameters and the three parameters of the Weibull Distribution. Within this work, the method of Probability Weighted Moments (PWM) given in [26] is applied to estimate the Weibull parameters.

As a matter of fact, the Weibull model proposed by Castillo et al. is a unified statistical methodology for modelling the fatigue damage. This methodology has been applied several times in order to estimate the fatigue life or to evaluate fatigue data. Some of these applications consider the strain [27], stress level and amplitude [28], mean stress and Walker model [29], multiaxial loading [30], scale or size effect [31,32].

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[1] Soltau, 22.06.1819 - Hannover, 21.03.1914. German engineer, Royal “Obermaschinenmeister” of the ”Niederschlesisch-Mährische” Railways in Frankfurt an der Oder.

[2] Also called Wöhler curves.

[3] Samples wich do not fail within a preestablished period of time. Statistically speaking, also called censored data type I.

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Table 1
Common models to represent the *S-N* curves.

Model	*S-N* curves equation
Basquin [13]	$\log N = A - B\log\Delta\sigma; \quad \Delta\sigma \geqslant \Delta\sigma_\infty$
Stromeyer [8]	$\log N = A - B\log(\Delta\sigma - \Delta\sigma_\infty)$
Bastenaire [2]	$N = \frac{A}{\Delta\sigma - E}\exp[-C(\Delta\sigma - E)] - B$
Ling and Pan [3]	$F = \sum_{i=1}^{n}\left\{\ln\sigma(S_i) + \frac{[\log N_i - \mu(S_i)]^2}{2\sigma^2(S_i)}\right\}$
Kohout and Věchet [9]	$\log\left(\frac{\Delta\sigma}{\Delta\sigma_\infty}\right) = \log\left(\frac{N+N_1}{N+N_2}\right)^b$

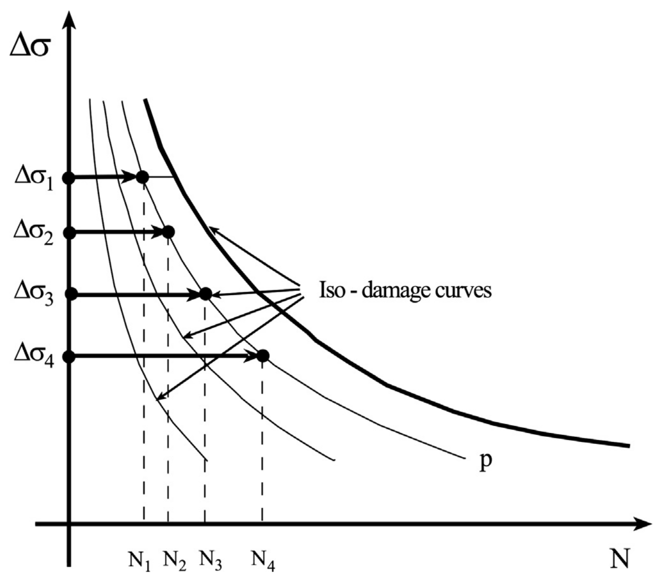

Fig. 1. Iso-damage *S-N* curves describing damage states. Representation of four different load histories at constant stress levels which lead to the same damage [23].

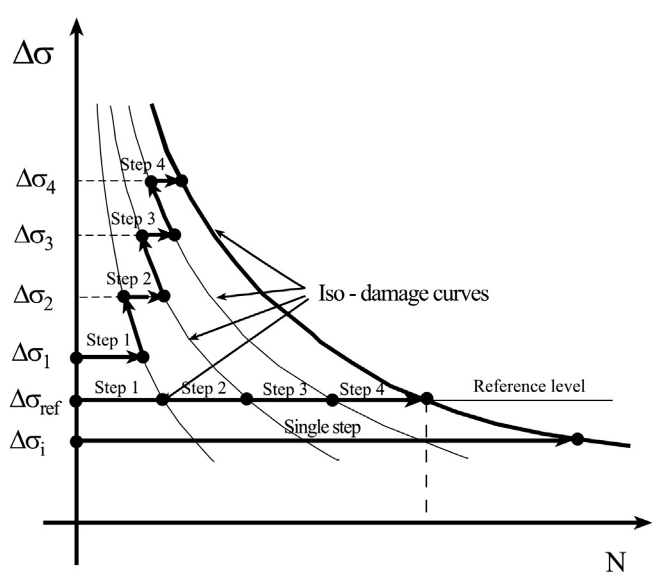

Fig. 2. Iso-damage *S-N* curves of multiple steps loading describing damage states. Representation of four different steps loadings at constant stress levels which lead to the same damage [23].

In this article, two applications are presented and the corresponding fatigue data include fatigue failures, runouts and subsequent retests from the runouts. These experimental data come from two different sources: (a) main girders from an antique steel bridge built in 1895, (b) welded specimens of steel S690QL.

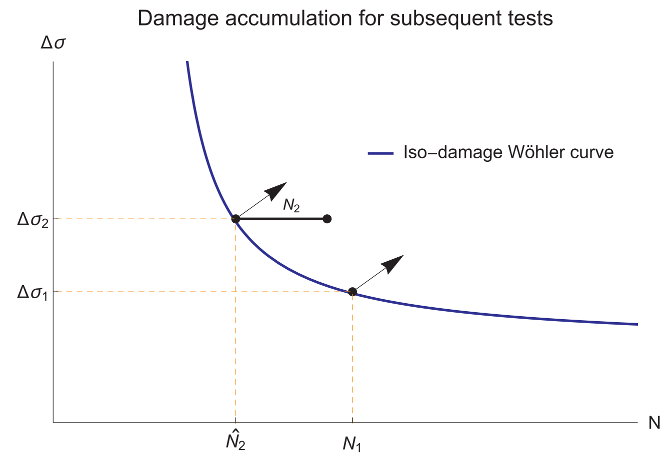

Fig. 3. Damage accumulation for a retest of a runout under higher loading. Both runouts have the same probability of fatigue failure.

According to the available experimental data, three types of data groups or samples can be obtained.

F: sample which contains only fatigue failures.
F-RO: sample which contains fatigue failures and runouts.
F-RO-RT: sample which contains fatigue failures, runouts and re-tested runouts.

The obtained results are presented through the comparison between the *S-N* curves given by every sample.

2. Weibull model

The three-parameter Weibull distribution $W(a,b,c)$ belongs to the family of extreme value distributions. The cumulative distribution function (CDF) also called life distribution or failure distribution of a random variable x which follows a $W(a,b,c)$ [33] is given by

$$F(x|a,b,c) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^c\right], \qquad x \geqslant a, \tag{1}$$

where

$a \in \mathbb{R}$: location parameter, minimum life or threshold.
$b > 0$: scale parameter or characteristic life.
$c > 0$: shape parameter or slope of $F(x|a,b,c)$.

Denoting $x = (\log N - B)(\log\Delta\sigma - C)$, and based on a three-parameter Weibull distribution $W(a,b,c)$, Castillo et al. propose the following equation to model the *S-N* curves:

$$Q(N,\Delta\sigma) = 1 - \exp\left\{-\left[\frac{(\log N - B)(\log\Delta\sigma - C) - a}{b}\right]^c\right\}, \tag{2}$$

where

$Q(N,\Delta\sigma)$: probability of failure.
$\Delta\sigma$: stress range during the fatigue test.
N: number of load cycles up to failure during the test.
B: threshold value of lifetime N.
C: endurance limit for $\Delta\sigma$.

The model given by Eq. (2) depends on two geometrical parameters B and C, and three Weibull parameters a, b and c, which should be estimated.