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Plasticity theory for the multiaxial Local Strain-Life Method

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ABSTRACT

This paper outlines a cyclic plasticity theory whose aim is to allow fatigue designers to make calculations for multiaxial loads in a way as similar as possible to what they do when using the well-known Local Strain methodology for uniaxial low–cycle fatigue problems. Thus we define concepts that translate to multiaxial loadings the intuitive methods for stress and strains calculations based on the use of the Ramberg-Osgood equation for the cyclic stress-strain curve, the adoption of Masing behavior with a factor-of-two assumption to model the hysteresis loops of the material from the cyclic curve and the invocation of the memory rule when hysteresis loops are "closed". The present theory is based upon the idea of distance between stress points and to calculate these distances we use the expression of the yield criterion of the material.

1. Introduction

The *local strain* or *strain-life* method constitutes nowadays a standard tool for low-cycle fatigue life calculations in many industries. It has been incorporated in commercial software [1,2] and is very well described in textbooks [3–5]. The use of the cyclic curve, the hysteresis loops, Neuber's rule and the memory effect is familiar, in the uniaxial load case, to most engineers concerned with low–cycle fatigue issues.

However, the extension of the Local Strain method to the multiaxial case has not achieved yet the desired degree of simplicity. It requires three main steps at least. The first one is the development of plastic flow rules which reproduce the way we operate with hysteresis loops, cyclic curves, memory effect and so on in the simple uniaxial case. The second step would be the development of multiaxial Neuber-type rules for dealing with inelastic strains at notches. This relies heavily on the use of a theory of plasticity and hence on the previous step. There are many proposals in this respect, starting with the pioneering work of Hoffmann and Seeger [6]. More recently, Glinka et al. [7] have obtained many important results. The third step is probably the most difficult and is the area where more work has been done so far: the multiaxial cycle counting and fatigue life criteria. There are too many of them to single any one out. A comparison of several criteria is provided in [8]. They need the stresses and strains as inputs and, therefore, they also depend on the two previous steps.

We are concerned here with the first step. We are trying to develop a theory of cyclic plasticity which allows fatigue designers to make calculations for multiaxial loads in a way as similar as possible to what they do when using the well-known Local Strain methodology for uniaxial low cycle fatigue problems. Thus we would like to develop a multiaxial theory where there is a method similar to the invocation of the memory rule when hysteresis loops are "closed", something which is conspicuously missing from current formulations of cyclic plasticity. Such a multiaxial memory rule may be used to define cycle counting procedures for multiaxial variable amplitude loading.

We have found it useful to base this theory on the idea of distance between stress points and to calculate these distances by using the expression for the yield criterion. The theory does not make use of yield or loading surfaces that move about in stress space, a common ingredient of existing cyclic plasticity theories. It uses the concept of distance in a stress space endowed with a certain metric which is determined from the yield criterion. The evolution and full mathematical details of the theory have been given elsewhere [9–14] and we would just like to give here a quick overview of the procedure. To keep the discussion at the simplest possible level, after presenting the general ideas of the theory, we restrict the treatment to the case of combined tension and torsion loading. The application of the equations to the analysis of non-proportional experiments is shown.

2. Plastic strains

The Local Strain method revolves around a simplified description of the stress-strain behavior. A very characteristic feature of

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Nomenclature

E	Young's modulus
$H()$	function describing isotropic hardening
$H^{-1}()$	inverse function of $H()$
k	yield stress in pure shear (torsion)
K	strength coefficient in the Ramberg-Osgood relationship
n	strain hardening exponent in the Ramberg-Osgood relationship
$\mathbf{n}$	unit normal to the surfaces $\lvert\boldsymbol{\sigma}\rvert = constant$ or $q = constant$
n_1, n_2	components of unit normal
q	effective distance between stress points after load reversals
dq	increment of the effective distance q
$q_0, q_1, \ldots, q_K, \ldots$	diameters of circles $C_0, C_1, \ldots C_K, \ldots$
dW_p	increment of plastic work
Y	yield stress in uniaxial tension (or compression)
γ^p	plastic shear (torsion) strain component
$d\gamma^p$	increment of plastic shear strain component
ε	axial strain component
ε^p	plastic axial strain component
$d\varepsilon^p$	increment of plastic axial strain component
$\boldsymbol{\varepsilon}^p$	plastic strain vector (linear form)
$\lvert\boldsymbol{\varepsilon}^p\rvert$	magnitude of the plastic strain vector
$d\boldsymbol{\varepsilon}^p$	increment of plastic strain vector (linear form)
$\lvert d\boldsymbol{\varepsilon}^p\rvert$	magnitude of the increment of the plastic strain vector
$d\lvert\boldsymbol{\varepsilon}^p\rvert$	increment of the magnitude of the plastic strain vector
θ	angle between stress vectors
σ	axial stress component
$\boldsymbol{\sigma}$	stress vector or stress point
$\lvert\boldsymbol{\sigma}\rvert$	magnitude of the stress vector
$d\lvert\boldsymbol{\sigma}\rvert$	increment of the magnitude of the stress vector
$\boldsymbol{\sigma}_0, \boldsymbol{\sigma}_1, \ldots, \boldsymbol{\sigma}_K, \ldots$	successive points of load reversal
$\sigma_0, \sigma_1, \ldots, \sigma_K, \ldots$	axial components of stress points $\boldsymbol{\sigma}_0, \boldsymbol{\sigma}_1, \ldots, \boldsymbol{\sigma}_K, \ldots$
$\boldsymbol{\sigma}_{c,1}, \boldsymbol{\sigma}_{c,2}, \ldots, \boldsymbol{\sigma}_{c,K}, \ldots$	centers of circles $C_1, C_2, \ldots C_K, \ldots$
τ	shear (torsion) stress component
$\tau_0, \tau_1, \ldots, \tau_K, \ldots$	shear components of stress points $\boldsymbol{\sigma}_0, \boldsymbol{\sigma}_1, \ldots, \boldsymbol{\sigma}_K, \ldots$
$\Phi()$	hardening modulus function. Derivative of $H^{-1}()$
$\phi()$	hardening modulus function after load reversal

the calculations of plastic strains in low–cycle fatigue problems is the clear distinction between loading and unloading. In the uniaxial case, one speaks of loading when the stress goes up in the cycle of applied stress and of unloading when it goes down. During the first quarter of the very first cycle, we "move" along the *cyclic curve* (dashed line in the lower part of Fig. 1) until unloading starts, marking the first point of load reversal (point A). We then "depart" from the cyclic curve and switch to the hysteresis loop. After a while moving along the descending branch of the hysteresis loop, another point of load reversal (point B) is reached, and we leave the current branch of the loop being traversed and start a new branch going up, and so on.

One of the key elements in the simulation of the $\varepsilon - \sigma$ behavior at a notch for variable amplitude loading is the correct application of the *memory effect* (see [3, chapters 12–14] and [5, chapter 9]), both for closing hysteresis loops and for switching the axes where the Neuber's hyperbolas are drawn for each load excursion. This is shown to occur in Fig. 1 as one moves, for example, from point D to point E. After reaching point E, the strain is then decreased to point F, following the path determined by the hysteresis loop shape. Upon re-loading, after reaching point EE′, E≡E′, the material continues to point A along the hysteresis path starting from point D, proceeding just as if the small loop E-F-E had never occurred. The same thing happens in the loop B-C-B. As we point out later on, this memory rule is a simplified representation of the so-called kinematic hardening.

As can be seen, the application of the memory effect depends on a precise control over the distance or separation, in terms of stress, between the successive points of load reversal. Thus, for example, when the stress is descending from C, the memory effect is invoked at B', where the *distance* between the current stress point and C becomes equal to the distance previously established between B and C. Distances between stress peaks and valleys are kept in a stack for comparison, and this kind of comparison (at the applied stress level) is really the basis of the cycle counting methods, such as the well-known Rainflow algorithm.

It is not at all clear how we can perform these checks in a multiaxial situation, where some of the components of stress may be increasing while others may be decreasing at the same time.

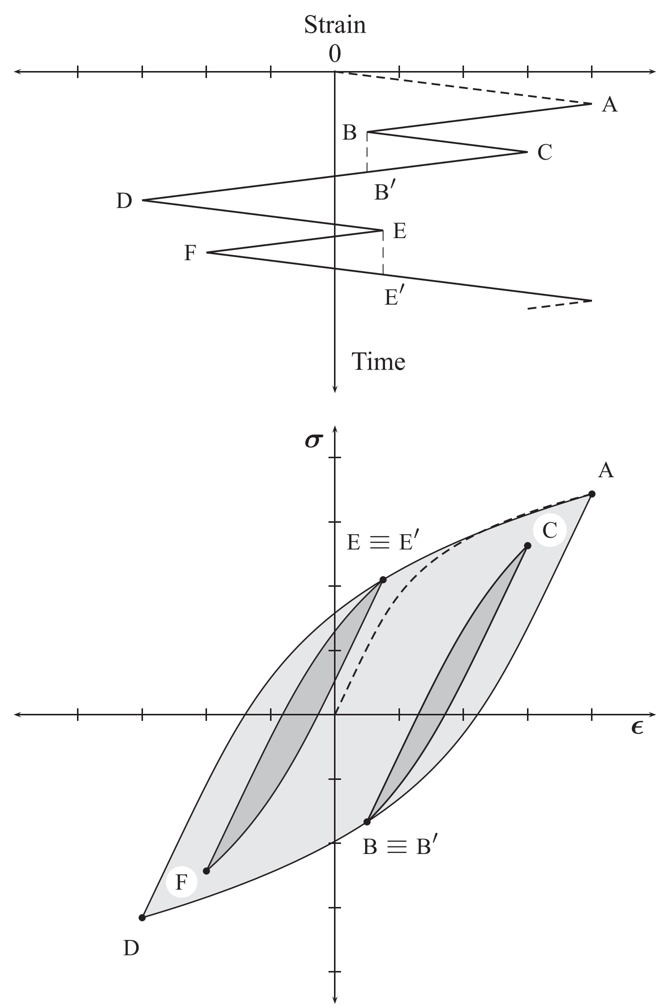

Fig. 1. Uniaxial memory effect.