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# On the consideration of normal and shear stress interaction in multiaxial fatigue damage analysis

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ABSTRACT

Due to the abundance of engineering components subjected to complex multiaxial loading histories, being able to accurately estimate fatigue damage under multiaxial stress states is a fundamental step in many fatigue life analyses. In this respect, the Fatemi-Socie (FS) critical plane damage parameter has been shown to provide satisfactory fatigue life correlations for a variety of materials and loading conditions. In this parameter, shear strain amplitude has a primary influence on fatigue damage and the maximum normal stress on the maximum shear plane has a secondary, but important, influence. Additionally, in order to preserve the unitless feature of strain, the maximum normal stress is normalized by the material yield strength. However, in examining some data from literature it was found that, in certain situations, the FS parameter can result in better fatigue life predictions if the maximum normal stress is normalized by the shear stress range on the maximum shear plane instead. These data include uniaxial loadings with large tensile mean stress, and some combined axial-torsion load paths with different normal-shear stress interactions. This modification to the FS parameter was investigated by using fatigue data from literature for 7075-T651 aluminum alloy and a ductile cast iron, as well as additional data from 2024-T3 aluminum alloy fatigue tests performed in this study.



## 1. Introduction

Although there are several different steps involved in the fatigue life estimation process [1], relating the variation of stresses and strains to the fatigue damage that occurs within a material is the most fundamental part of any fatigue life analysis. Due to the abundance of engineering components subjected to multiaxial loading histories, being able to accurately estimate fatigue damage under multiaxial stress states is especially important. Although there are many different methodologies for damage calculation, there are certain characteristics that any fatigue damage parameter should possess in order to help ensure that it is robust and generally applicable to a wide variety of fatigue life analyses. These include the ability to account for varying stress states, mean stresses, changes in material constitutive behavior due to cyclic and/or non-proportional hardening, and other effects such as load interaction and path dependence [1,2]. A parameter that can successfully incorporate all of these features has the best chance of being successful in even the most complex fatigue loading conditions, which often exist in multiaxial service loading applications.

Early multiaxial fatigue damage parameters, sometimes referred to as classical approaches, focus on computing an equivalent stress/strain quantity through the extension of static yield criteria. This equivalent stress/strain is then considered to be equal, in terms of fatigue damage, to a uniaxial loading of the same magnitude. A corresponding fatigue life can then be calculated from a uniaxial fatigue curve. Examples of some common multiaxial fatigue criteria include von Mises equivalent stress or strain and maximum shear stress or strain for ductile behaving materials, and maximum principal stress or strain for brittle behaving materials. In the event that mean or residual stresses are present, an additional relation is needed to compute an equivalent mean stress for use in a uniaxial mean stress correction model [2,3].

Although these classical approaches are simple in concept and easy to implement, they typically do not reflect the damage mechanisms at work within the material [2]. It has been shown in multiple studies that effective stress or strain parameters such as von Mises fail to bring together fatigue data even for the two simple cases of uniaxial and pure torsion loading, for example in [4,5]. Additionally, they cannot account for increased fatigue damage under non-proportional loading conditions [6–9] and have been shown to result in inadequate correlation of fatigue data from tests performed under different ratios of axial to shear stress [6,7]. Consistent with these findings, equivalent stress- and strain-based

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**Nomenclature**

| | |
|---|---|
| $b$ | axial fatigue strength exponent |
| $b_o$ | shear fatigue strength exponent |
| $c$ | axial fatigue ductility exponent |
| $c_o$ | shear fatigue ductility exponent |
| $E$ | modulus of elasticity |
| $G$ | shear modulus |
| $k$ | FS parameter material constant |
| $N_f$ | cycles to failure |
| $R$ | minimum to maximum stress ratio |
| $\gamma'_f$ | shear fatigue ductility coefficient |
| $\gamma_{a,e}$ | elastic shear strain amplitude |
| $\gamma_{a,p}$ | plastic shear strain amplitude |
| $\Delta\gamma_{max}$ | maximum shear strain range |
| $\Delta\tau$ | shear stress range |
| $\varepsilon'_f$ | axial fatigue ductility coefficient |
| $\varepsilon_{a,e}$ | elastic strain amplitude |
| $\varepsilon_{a,p}$ | plastic strain amplitude |
| $\lambda$ | nominal stress ratio ($\tau/\sigma$) |
| $\nu_e$ | elastic Poisson's ratio |
| $\nu_p$ | plastic Poisson's ratio |
| $\sigma'_f$ | axial fatigue strength coefficient |
| $\sigma_a$ | normal stress amplitude |
| $\sigma_{n,max}$ | maximum stress normal to maximum shear plane |
| $\sigma_{vm,a}$ | von Mises equivalent stress amplitude |
| $\sigma_y$ | tensile yield strength |
| $\tau_a$ | shear stress amplitude |
| $\tau'_f$ | shear fatigue strength coefficient |

analysis approaches were also found to result in relatively poor fatigue life correlations for the 2024-T3 aluminum alloy tested in the current study [10,11].

In order to overcome the shortcomings of classical multiaxial fatigue damage parameters, significant effort in the last few decades has been put into developing more sophisticated damage parameters which reflect the actual damage mechanisms of the fatigue failure process [1,2,12]. Chief among these are critical plane approaches, which build on the observation that fatigue cracks tend to initiate on preferred planes within a material. Critical plane approaches are typically based on the idea of crack initiation occurring on or around either the maximum principal plane or the maximum shear plane(s). As a result, these approaches have the added benefit of being able to predict failure plane orientation, which is useful information if a subsequent crack growth analysis is to be performed.

Perhaps one of the most difficult aspects to address in multiaxial fatigue damage calculation is how the interaction of shear and normal stress/strain components affects fatigue damage mechanisms. Normal-shear stress/strain interaction can occur in many forms including: the interaction between shear and normal stress components on the maximum shear plane (even under uniaxial loading, e.g. mean stress effects), different ratios of applied shear to normal stress amplitude, and time/load path dependent interaction effects between normal and shear stress/strain components. As a result, many critical plane damage parameters have been proposed over the years which compute fatigue damage based on various combinations of shear and normal stress and strain components, e.g. [13–21]. However, few have been shown to provide consistently accurate fatigue life predictions for a wide variety of materials and loading conditions.

### 1.1. Fatemi-Socie damage parameter

A popular critical plane-based parameter for computing multiaxial fatigue damage in materials exhibiting shear failure mechanisms is the Fatemi-Socie (FS) parameter [17]. This parameter was formulated based on the idea that while alternating shear strain is the primary driving force behind fatigue crack initiation, the maximum normal stress on the shear plane also affects the nucleation and growth of small cracks by influencing the amount of friction and interlocking between opposing crack faces.

This concept is supported through fatigue data generated by Socie and Shield [22] under several different multiaxial loading paths for Inconel 718. These paths all featured the same maximum shear and normal strain amplitudes, but different levels of mean normal stress on the maximum shear strain plane. By studying crack growth data for cracks up to around 2 mm in length, the crack growth rate was found to increase with increasing maximum normal stress, leading to shorter observed fatigue lives. The effect was found to be relatively small for cracks on the order of the material grain size, but increased with crack length.

Similarly, when studying the effects of static mean stress on shear-mode crack growth in tubular specimens of 1045 steel, Kaufman and Topper [23] found that by increasing the tensile normal stress on the maximum shear plane, fatigue life continually decreased until a critical level of mean stress was applied. Conversely, by simultaneously applying axial and hoop stresses, the effect of compressive mean stress, normal to both maximum shear planes, was found to increase fatigue life. After studying crack front and fracture surface asperity profiles, this behavior was again attributed to varying levels of friction and mechanical interlocking between opposing crack faces. These findings are consistent with those reported in other studies, e.g. [24, 25], as well. Therefore, the inclusion of the maximum normal stress per cycle in the FS parameter not only predicts an increase in fatigue damage due to non-proportional loading, but it also accounts for the effects of mean stress in a manner that holds physical significance, consistent with these experimental results.

The FS parameter predicts fatigue life in terms of shear fatigue properties based on the following equation:

$$\frac{\Delta\gamma_{max}}{2}\left(1+k\frac{\sigma_{n,max}}{\sigma_y}\right)=\frac{\tau'_f}{G}(2N_f)^{b_o}+\gamma'_f(2N_f)^{c_o} \tag{1}$$

where $\Delta\gamma_{max}$ is the maximum range of shear strain experienced on any plane, $\sigma_{n,max}$ is the maximum normal stress occurring on the same plane for the cycle of interest, $\sigma_y$ is the material yield strength, and $k$ is a material dependent parameter reflecting the influence of normal stress on fatigue damage. The maximum normal stress is normalized by yield strength as a means of preserving the unitless feature of strain.

The right-hand side of Eq. (1) represents the shear strain-life curve for the material under consideration. In the event that shear fatigue properties are not available for damage calculation, the right side of this equation may alternatively be expressed in terms of uniaxial fatigue properties as follows [3]:

$$\frac{\Delta\gamma_{max}}{2}\left(1+k\frac{\sigma_{n,max}}{\sigma_y}\right)=\left[(1+\nu_e)\frac{\sigma'_f}{E}(2N_f)^{b}+(1+\nu_p)\varepsilon'_f(2N_f)^{c}\right]\times\left[1+k\frac{\sigma'_f}{2\sigma_y}(2N_f)^{b}\right] \tag{2}$$