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# A multiscale approach for modeling fatigue crack growth in concrete

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ABSTRACT

A linearized stress intensity factor (SIF) is derived for concrete through a multiscale approach by considering the predominant process zone mechanisms such as aggregate bridging and microcracking. This is achieved by considering a bridging zone and a microcrack at the macrocrack tip. The bridging zone resists the crack growth through aggregate bridging mechanism. The SIF thus derived is further used in developing an analytical model which predicts the entire crack growth curve for plain concrete by making use of the concepts of dimensional analysis and self similarity in conjunction with the human population growth model. This model is validated using experimental data reported on normal strength, high strength and self consolidating concrete. Through sensitivity analyses it is shown that the specimen size plays an important role in the fatigue crack growth process of concrete.



## 1. Introduction

Failure of structural components occur due to several reasons, such as fatigue, fracture, corrosion, creep etc which affects the life of the structure. Fracture failure occurs when material imperfections, in the form of internal flaws or microcracks grow into a macrocrack leading to the separation of the component into two or more parts under the influence of load. Fracture mechanics provides the basic mathematical theory and helps in understanding the failure of materials based on the assumption that there exist a crack in the component [1]. The main factors affecting the fracture of a material are stress concentration, rate of loading, temperature, thermal shock etc. The presence of crack in the material triggers failure due to increase in the stress near the crack tip. Fracture failure is generally characterized by the parameter called stress intensity factor (SIF), which describes the state of stress in the close vicinity of the crack tip. Irwin defined the fundamental concept of SIF ($K_I$) and the critical SIF ($K_{Ic}$) which is a material property to provide a criterion for fracture in terms of the asymptotic stress and displacement fields around a crack in a linear elastic solid. This is further used in design and analysis by arguing that the material can withstand crack tip stresses up to a critical value of stress intensity ($K_{Ic}$), beyond which the crack propagates rapidly leading to failure.

Materials which exhibit a moderate strain hardening behavior before reaching its tensile strength and there after followed by a tension softening behavior is known as quasi-brittle [2]. The ultimate failure of such materials will be triggered by various local mechanisms occurring in the material near the crack tip region. Concrete being a quasi-brittle material is treated as homogeneous from the design perspective even though it consist of a mixture of cement paste and water that binds the inert aggregates into a rock-like mass as the paste hardens through chemical reaction of cement with water. The complex behavior of concrete due to the presence of these heterogeneities can be well understood by studying their individual contributions and their mutual interactions. Understanding and modeling of concrete behavior not only provides a proper design but also helps in developing new cement based materials.

In concrete, there exists an intermediate zone between the cracked and uncracked portion, which is defined as the fracture process zone (FPZ). This zone offers resistance to the crack growth due to the presence of various toughening mechanisms such as aggregate bridging, microcracking, crack branching etc. Nirmalendra and Horri [3] have reported that the major mechanisms responsible for the softening behavior of quasi-brittle materials are microcracking and aggregate bridging. In a microcracking zone, initiation of microcracks and their growth are dominant, which is initiated due to stress concentration near the macrocrack. The bridging zone is a part of a macrocrack along which stress is transmitted through aggregate bridging [4]. As the crack propagates, these microcracks coalesce to give continuity to the already existing crack. Careful analysis of the fracture process zone helps in predicting the behavior and propagation of crack and ultimate failure in cementitious materials.

The presence of FPZ in quasi-brittle materials restricts the use of linear elastic fracture mechanics (LEFM) approach due to the

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**Nomenclature**

| | |
|---|---|
| $a$ | crack length |
| $a_c$ | critical macrocrack length |
| $B$ | thickness of beam |
| $b$ | length of bridging zone |
| $C$ and $m$ | fatigue law parameter |
| $C_s$ | coefficient of sensitivity |
| $C_v$ | coefficient of variation |
| $D$ | depth of beam specimen |
| $d_{a,max}$ | maximum aggregate size |
| $E$ | elastic modulus of concrete |
| $E^{Interface}$ | interfacial elastic modulus |
| $E_m$ | elastic modulus of mortar |
| $E_{ca}$ | elastic modulus of coarse aggregate |
| $E_{cp}$ | elastic modulus of cement paste |
| $E_{fa}$ | elastic modulus of fine aggregate |
| $E_{micro}$ | elastic modulus at micro scale |
| $E_{macro}$ | elastic modulus at macro scale |
| $E_{eff}$ | effective elastic modulus at the interface |
| $F_u$ | peak load |
| $G_F$ | size dependent fracture energy |
| $G_F^{Interface}$ | interfacial fracture energy |
| $K_{If}$ | fracture toughness |
| $K_{IC}$ | size-dependent equivalent fracture toughness |
| $K_{IC}^{Interface}$ | interfacial fracture toughness |
| $K_{IC}^{m}$ | fracture toughness of mortar |
| $K_{IC}^{cp}$ | fracture toughness of cement paste |
| $K_I$ | mode I stress intensity factor |
| $l$ | critical microcrack length |
| $l_p$ | length of process zone |
| $N$ | number of cycles |
| $P$ | applied load |
| $R$ | stress ratio |
| $S$ | span of beam |
| $V_f(ca)$ | volume fraction of coarse aggregate |
| $V_f(fa)$ | volume fraction of fine aggregate |
| $V_f(m)$ | volume fraction of mortar |
| $V_f(cp)$ | volume fraction of cement paste |
| $\alpha$ | relative crack length (a/D) |
| $\beta^*$ | half angle of microcrack |
| $\delta^{micro}$ | microscopic crack opening displacement |
| $\delta^{macro}$ | microscopic crack opening displacement |

absence of sharp crack tip. The energy dissipated is not available for crack propagation due to the redistribution of stress in the process zone. The size of this FPZ can be commensurate with that of most structural elements [5]. When the size of the FPZ is substantially lower than the characteristic dimension ($D$) of the structure, crack extension can be quantified using LEFM procedures, otherwise, a nonlinear fracture mechanics (NLFM) analysis needs to be performed. Several investigations [6–8] have reported that the fracture properties of concrete cannot be characterized using linear elastic fracture mechanics (LEFM). The fracture and fatigue properties of such materials can be modeled using nonlinear mechanistic models, but these make the analysis quiet cumbersome. Hence linearized models that include the physics of fracture process zone need to be developed. This could be achieved by including the mechanisms at different scales through a multiscale approach.

It is well known that concrete structures such as bridges, roads and airfields undergo a gradual destruction as it is subjected to fatigue loading due to the inhomogeneities and the structural defects present in the material. Generally, the fatigue failure in structural components takes place under repetitive load of magnitude much lower than their static load capacity. The fatigue failure is defined as a progressive, permanent change in the internal structure of the material in the form of a microcrack which coalesces to form the macrocrack. The fatigue crack propagation rate was estimated by Paris and Erdogan [9] using the fracture mechanics approach where in the crack growth increment per load cycle was defined as a function of the stress intensity factor range. Due to the presence of large process zone at the crack tip in quasibrittle materials like concrete, the fracture properties derived based on LEFM approach gives erroneous results [10]. Attempts were made by various researchers [11–15] to characterize the fatigue behavior of concrete by modifying the well known Paris law [9]. Later, the concepts of dimensional analysis and incomplete self similarity proposed by Barenblatt [16] were used to characterize the fatigue behavior of concrete. Various researchers [17–19] used this concept for predicting the fatigue crack propagation law of concrete by including several material parameters.

In this study, an LEFM based method is attempted to describe failure in quasi-brittle material like concrete using a multiscale approach by modifying the definition of SIF by relating the crack opening displacement at the macroscale and the microscale. This modified SIF based on LEFM approach includes the contributions from bridging stress which occurs due to aggregate bridging at the mesoscale and microcracking occurring at microscale which is derived by considering a free-free boundary condition along the microcrack surface. The nonlinear behavior of the FPZ is captured by considering the various toughening mechanisms such as aggregate bridging and micro cracking occurring at meso and micro length scales. This is followed by developing a fatigue crack growth model using the analogy of the human population growth in conjunction with the concepts of dimensional analysis and self similarity. The crack growth law is developed by making use of the modified SIF. The proposed model is validated using the experimental data taken from the literature. Further, a probabilistic analysis is carried out to determine the sensitivity of each of the various parameters that has been considered in the crack growth model.

## 2. Determination of modified stress intensity factor

An expression for stress intensity factor (SIF) is derived based on LEFM approach by considering the material behavior in its lower scale for the case of a notched plain concrete beam, whose geometry and loading pattern are shown in Fig. 1. In cementitious

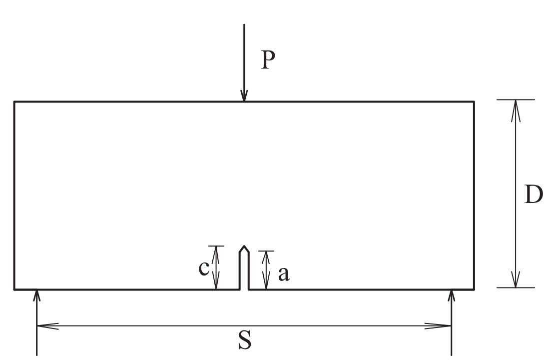


**Fig. 1.** Geometry of three-point bend beam specimen.