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A one-parameter nonlinear fatigue damage accumulation model

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ABSTRACT

A nonlinear model for fatigue damage accumulation under variable amplitude loading is presented. The known assumption that the isodamage curves are converging at the knee point of the S-N curve of the material, has been adopted. The proposed model does only require one parameter, in addition to the S-N curve of the material, in order to calculate the remaining fatigue life. A single value for the parameter has been found to give satisfying agreement with experimental data for four arbitrary selected steels, indicating a general trend.

1. Introduction

In order to calculate the cumulative fatigue damage of steel structures subjected to variable amplitude loading, design standards like Eurocode 3 [1] and DNVGL-RP-C203 [2] still recommend the use of the Palmgren-Miner rule [3,4], which was stated mathematically in 1945 [5]. This rule is also commonly used in the European automotive industry [6]. However, it has been known for a long time that this linear damage rule predicts a longer fatigue life than normally experienced if the loading amplitude is decreasing, and a shorter life if the loading amplitude is increasing [5,7–9]. The conservatism of the Palmgren-Miner rule is therefore questionable. Probabilistic approaches to overcome this shortcoming are discussed in [10–12]. Because of this problem, numerous cumulative fatigue damage rules have been proposed [5,7,8,13–23].

One model, which has shown excellent agreement with experimental results, is the double linear damage rule, developed by Manson et al. during the 1960s and ’70s [9,13,14]. This rule divides the fatigue life into two phases; Phase I and Phase II, in which different mechanisms are assumed to dominate. The double linear rule assumes that the behaviour within each of these two phases may be modelled by a linear rule, thereby preserving some of the simplicity of the Palmgren-Miner rule.

The two phases were originally considered to represent crack initiation and crack growth, but Manson and co-workers did later abandon this phrasing for their model [13]. However, other models differentiating between crack initiation and crack growth were developed, like the double exponential law by Miller and Zachariah [15] from 1977. These models typically require tests to be performed in order to determine the duration of each of these two stages at different stress levels.

Also developed in the 1970s was the concept of isodamage lines [5]. Subramanyan provided a simple model [16] in 1976, in which the isodamage lines were assumed to converge at the knee point of the S-N curve, as will be explained in Section 2.2. An alternative approach of the concept of isodamage lines was proposed by Hashin and Rotem [17] in 1978. In this work they assumed isodamage lines converging at the point where the S-N curve intersects the S-axis. Since these alternative isodamage lines become invalid at low stress amplitude values [17], the selection of Subramanyan’s model is preferable. Additional confirmation of the validity of the assumption of Subramanyan’s model can be found in [18], indicating the existence of isodamage lines converging at the knee point, by considering the evolution of surface hardness during fatigue damage accumulation. The model containing isodamage lines converging at the knee point shows a clear improvement from the Palmgren-Miner rule, but is still slightly nonconservative [14].

As shown in the reviews by Fatemi and Yang [5] and Santecchia et al. [19], many other cumulative damage rules have been proposed over the years. In order to account for more of the underlying mechanisms, the simplicity of the double linear rule and Subramanyan’s model is often lost, but still none of them has reached universal acceptance yet [19]. Many of the models require some material-dependent coefficient to be determined through extensive testing, e.g. [20]. Such test data or testing facilities may not be available to a design engineer, causing fatigue calculation by these models to be difficult.

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Nomenclature

a	coefficient relating D_i to the evolution of material parameters
b	model parameter
C_i	experienced cycle ratio, or estimated remaining cycle ratio, at load step i
D, D_i	accumulated damage up to and including load step i
i	load step number
$m(\sigma)$	coefficient
N	fatigue life
N_e	number of cycles at the knee point of the S-N curve
N_i	fatigue life for cyclic loading with constant stress amplitude σ_i
N_{irem}	remaining number of cycles with stress amplitude σ_i until fatigue failure
n, n_i	number of load cycles applied at load step i
n_{ij}	the number of cycles of stress amplitude σ_j which causes the same fatiguedamage as n_i cycles of stress amplitude σ_i
$q(\sigma_i)$	the exponent in the proposed formula for D_i
α_i	coefficient in Subramanyan's model
θ_f	angle between the isodamage line for zero damage, and the S-N curve
θ_i	angle of the isodamage line through the point (n_i, σ_i) in the S-N diagram
$\Delta\sigma$	stress range $= 2\sigma$
σ	stress amplitude
σ_e	endurance limit (fatigue limit), given as stress amplitude
σ_i	stress amplitude at load step i
σ_s	stress amplitude at intersection between S-N curve and σ-axis

However, some models have recently been developed, which do not require extensive testing. One example is the damage curve approach by Gao et al. [21], based on a similar approach by Manson et al. [13]. Furthermore, a simple model, taking the mean stress of the cyclic loading into account, has been developed by Shang and Yao [22], based on the continuum fatigue damage theory. Another recent damage rule is the sequential law, developed by Mesmacque et al. [7,23]. This rule also does not need any experimentally determined constant, apart from the full S-N curve of the material, and has been applied in order to estimate the remaining fatigue life of railway bridges [24]. However, significant deviation between the sequential law and real fatigue life has been observed [25]. Therefore, a similar rule, where the plastic meso-strain is used in the damage indicator, instead of the von Mises stress, has also been developed [25].

In the present work, a fatigue damage accumulation model based on isodamage curves is proposed. The model is intended to be simple to use for practicing engineers, while also being more accurate than comparable models. The shape of the isodamage curves is based on experimental data for two-step fatigue tests of SAE 4130 steel, reported by Manson et.al. [26]. The proposed model is then compared to the Palmgren-Miner rule, Subramanyan's model, the double linear damage rule, Shang and Yao's model and the sequential law, using experimental data for C-35 steel [16], P355NL1 steel [27] and 300 CVM steel [26].

2. Fatigue damage accumulation

2.1. Isostress lines

Let us assume a constant stress amplitude (σ = const.). The elementary percentage of the damage increment, dD/D, should be proportional to the elementary percentage of the increment of the loading cycles, dn/n:

$$\frac{dD}{D} = m(\sigma)\frac{dn}{n} \tag{1}$$

where $m(\sigma)$ is a parameter depending on the stress amplitude.

Integration of Eq. (1) yields:

$$\int \frac{dD}{D} = m(\sigma)\int \frac{dn}{n} + C \tag{2}$$

or

$$\ln D = m(\sigma)\ln n + C \tag{3}$$

For $n = N$, failure of the material is occurring. Therefore, the damage function takes the value $D = 1$. Taking into account the above boundary condition, the constant of integration, C, can be obtained by Eq. (3):

$$0 = m(\sigma)\ln N + C \tag{4}$$

or

$$C = \ln N^{-m(\sigma)} \tag{5}$$

With the aid of Eqs. (5) and (3) yields:

$$\ln D = \ln n^{m(\sigma)} + \ln N^{-m(\sigma)} \tag{6}$$

or

$$D = \left(\frac{n}{N}\right)^{m(\sigma)} \tag{7}$$

The above functional form for damage accumulation is illustrated in Fig. 1, and has been proposed by many researchers, e.g. [13,21,28].

2.2. Isodamage lines

Subramanyan's model for cumulative fatigue damage [16] is based on the assumption that the S-N curve of a material corresponds to a state of 100% fatigue damage, while combinations of stresses and cycles (n_i, σ_i) below the endurance limit (fatigue limit) correspond to a state of 0% fatigue damage. In between these

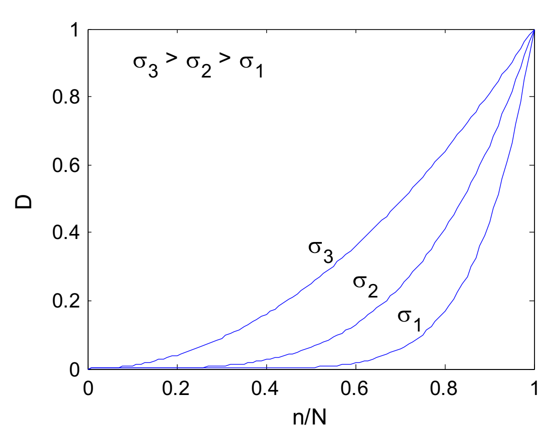

Fig. 1. Schematic demonstration of isostress curves according to Eq. (7).