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# Fatigue criterion improvement of Gough and Nishihara & Kawamoto to predict the fatigue damage of a wide range of metallic materials

Leila Khalij *, Emmanuel Pagnacco, Renata Troian

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ABSTRACT

An improved multiaxial high-cycle model based on the Gough and Nishihara & Kawamoto approaches is presented. The proposal is a compromise between the measure of the equivalent shear stress amplitude, obtained from a prismatic convex hull method, and the amplitude of the first stress invariant. This makes the criterion an effective approach to predict quickly the failure initiation for wide range of materials (from ductile metals to extremely brittle metals). The results obtained from our proposal are compared to experimental data reported in the literature.



## 1. Introduction

The fatigue life assessment of components and structures in service requires taking into account the stress state multiaxiality. In the past decades, for high-cycle fatigue or unlimited endurance, numerous studies have attempted to develop stress-based damage criteria. Among them we can mention criteria based on the second invariant amplitude of the deviatoric stress tensor (or equivalent shear stress amplitude) [1–5]. Most of them are based on the Von Mises equivalent stress and require a measure of its amplitude to retrieve the damage information (through S-N curves, for example).

In the plasticity field, the equivalent stress of von Mises is in good agreement with experimental tests on many ductile materials such as copper, aluminum or mild and alloy steels [6,7]. It is similar in fatigue field but this equivalent stress and therefore the related fatigue criterion are not adapted for hard or brittle steels. In the present study, basing on Nishihara and Kawamoto [9] and Gough [10] criteria, that are in agreement with experimental data for a wide range of materials [8], we generalized the criteria formulated for fully reversed bending-torsion tests to the plane stresses. In addition, a correction considering the effect of the hydrostatic stress is included in the model in order to extend the study to extremely brittle materials [9–12].

Phase shift between the two normal stresses when components or structures are submitted to a fatigue biaxial loading is not implicit in the new formulation. However, for non-proportional stresses with a time-evolution of the principal directions [13,14], the phase shift has an effect on the fatigue strength. This problem can be handled with elliptic or prismatic convex hull methods [15–17] which generalize the one proposed by Li et al. [18]. The advantage of the defining an amplitude measure by the hull methods is that a critical plane or a failure plane [11,19] doesn't have to be considered for the calculations. Therefore, a singular behavior of the damage function is not observed considering the in-quadrature phase between stress components [20].

In this work, we consider the "Prismatic Hull in the Principal coordinate System (PHPS)" method developed previously in [13]. It leads to an analytic measurement of the equivalent shear stress amplitude from sinusoidal stress components. The principal directions of the stress path in a 5-dimensional space of deviatoric stress is investigated. The measure of the equivalent shear stress amplitude is then directly evaluated from the semi-axes of the prismatic hull that circumscribes the stress path. The resulting expression is insensitive to the phase shift between normal and shear stresses but depends on the phase shift between two normal stresses, as is demonstrated in [13,21].

A numerical study is performed to point out the trends of the proposed criterion and confront them with the relevant experimental results of the biaxial tension, bending-torsion and plane stress (2D-stresses) collected from various sources.

* Corresponding author.
*E-mail address:* leila.khalij@insa-rouen.fr (L. Khalij).

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**Nomenclature**

| | |
|---|---|
| $\mathbf{E}_5$ | five components stress space |
| $\underline{\mathbf{I}}_d$ | identity matrix |
| $I_1$ | first invariant of the stress tensor |
| $J_2$ | second invariant of the stress deviator |
| $\sqrt{J_{2,a}}$ | equivalent shear stress amplitude |
| $k$ | biaxial stress ratio |
| $p_1(t),\ p_2(t),\ p_3(t)$ | principal stresses |
| $r$ | ratio of fatigue limits |
| $R$ | stress ratio |
| $\mathbf{s}(t)$ | stress vector |
| $s_{eq}(t)$ | equivalent stress |
| $s_{xx}(t),\ s_{yy}(t)$ | normal stress components |
| $s_{xy}(t)$ | shear stress component |
| $s_y$ | static tensile yield stress |
| $s_{-1},\ \tau_{-1}$ | fully reversed fatigue limits in tension (or bending) and in torsion |
| $s_0,\ \tau_0$ | fatigue limits under pulsating tensile and pulsating torsional stresses |
| $T$ | period of time |
| $\underline{\mathbf{V}}_S$ | mean square matrix |
| $\phi_{yy}$ | phase shift between normal stress components |
| $\omega$ | angular frequency |
| $\bullet(t)$ | time function |
| $\bullet_{,a}$ | amplitudes over time of $\bullet$ |
| $\bullet_{,m}$ | means over time of $\bullet$ |
| $\mathrm{E}[\bullet]$ | mean operator |
| $\max\limits_{t\in T}[\bullet]$ | maximum over time of $\bullet$ |
| $\min\limits_{t\in T}[\bullet]$ | minimum over time of $\bullet$ |
| $\bullet^{\mathrm{T}}$ | transpose of matrix $\bullet$ |

## 2. Background

As crack initiation occurs at the surface in fatigue, a plane stress state is chosen and the sinusoidal stress components are defined by:

$$s_\bullet(t) = s_{\bullet,m} + s_{\bullet,a}\sin(\omega t - \phi_\bullet) \tag{1}$$

$s_{\bullet,a} = \frac{1}{2}\left(\max\limits_{t\in T}[s_\bullet(t)] - \min\limits_{t\in T}[s_\bullet(t)]\right)$ and $s_{\bullet,m} = \frac{1}{2}\left(\max\limits_{t\in T}[s_\bullet(t)] + \min\limits_{t\in T}[s_\bullet(t)]\right)$ are respectively the amplitude and the mean of stress components, $\phi_\bullet$ is the phase shift between these components and $\omega$ is the angular frequency defined on a period $T$. Components $\bullet$ are $xx$ and $yy$ for normal stresses and $xy$ for shear stress.

A yield criterion is introduced in order to compare the maximum of an instantaneous equivalent stress, denoted $s_{eq}(t)$, to a threshold stress value. This threshold, initially denoted $s_c$, is the yield stress in a quasi-static analysis. A generic form is given by the following inequality:

$$\frac{\max_t[s_{eq}(t)]}{s_c} \leqslant 1 \tag{2}$$

Moreover, others alternatives involving the maximum of the first invariant of the stress tensor (denoted $I_1(t)$) can be found in the literature [22]:

$$\frac{\max_t[s_{eq}(t)]}{s_c} + \beta\left(\frac{\max_t[I_1(t)]}{s_c}\right) \leqslant 1 \tag{3}$$

where $\beta$ is a material parameter and:

$$I_1(t) = s_{xx}(t) + s_{yy}(t) \tag{4}$$

High cycle fatigue equivalent stress approaches can be viewed as an extension of the static yield criteria to fatigue. In order to apply the inequalities (2) or (3) to the high cycle fatigue computation, the threshold $s_c$ is related to the fatigue limit and $\beta$ is obtained from Eq. (3) using fully reversed uniaxial fatigue tests in tension, bending or torsion. Additionally, the first invariant of the stress tensor allows involving the hydrostatic stress effect in the formulation.

Regarding the fully reversed sinusoidal stress components ($s_{\bullet,m} = 0$ in Eq. (1)), the maximum of $s_{eq}(t)$ is quantified as an amplitude such as $\max_t[s_{eq}(t)] = s_{eq,max} = s_{eq,a}$. This quantity allows to introduce the equivalent shear stress amplitude $\sqrt{J_{2,a}} = \frac{s_{eq,a}}{\sqrt{3}}$, the main parameter of many stress based multiaxial fatigue criteria [1–5]. The von Mises stress is commonly used in determining the expression of $\sqrt{J_{2,a}}$. An alternative obtained from the prismatic hull in the principal coordinate system method [13] is presented in the next subsection. Then, the expressions of inequalities (2) and (3) are discussed for fatigue in the next subsections for general plane stress state. In these subsections, only the zero-mean stresses are considered.

### 2.1. Measure of the equivalent shear stress amplitude from a prismatic hull method

It is now common to introduce a suitable change of variables reducing the stress space from six to five components [13,23] to express the equivalent stress in terms of a stress deviator norm. Thus, let us consider a vector $\mathbf{S}(t)$ with five components in the space denoted $\mathbf{E}_5$. The change of variable over the stress is ensured by a transition matrix $\underline{\mathbf{P}}$ as:

$$\mathbf{S}(t) = \underline{\mathbf{P}}.\mathbf{s}(t) \tag{5}$$

where $\mathbf{s}(t)$ is the stress vector defined by six components. The equivalent shear stress calculation is deduced by:

$$\sqrt{J_2(t)} = \sqrt{\mathbf{S}^{\mathrm{T}}(t)\mathbf{S}(t)} \tag{6}$$

The tip of $\underline{\mathbf{S}}(t)$ describes in $\mathbf{E}_5$ a curve corresponding to the stress path. The measure of the equivalent shear stress, denoted $\sqrt{J_{2,a}}$, can be then defined from a hull that encloses the stress path. Various approaches have been proposed in recent years. Among them, we can mention elliptic convex hull methods enclosing the stress path in $\mathbf{E}_5$ [15,16] or prismatic hull methods [13,17]. Other alternative is the moment of inertia method proposed by Meggiolaro et al. [24], or, in the following, the prismatic hull defined in the principal coordinate system (denoted PHPS by the authors) [13].

In the case of the in-plane stress, the three-dimensional stress space $\mathbf{E}_3$ is considered. The change of a variable over the stress in $\mathbf{E}_3$ is given by:

$$\mathbf{S}(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ S_4(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{xx}(t) \\ s_{yy}(t) \\ s_{xy}(t) \end{bmatrix} \tag{7}$$

In this case, the transition matrix $\underline{\mathbf{P}}$ is:

$$\underline{\mathbf{P}} = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$