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Epistemic uncertainty quantification in metal fatigue crack growth analysis using evidence theory

Hesheng Tang [a,*], Dawei Li [b], Jingjing Li [b], Songtao Xue [b]

[a] State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, 1239, Siping Rd., Shanghai, China
[b] Research Institute of Structural Engineering and Disaster Reduction, Tongji University, 1239, Siping Rd., Shanghai, China

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ABSTRACT

Uncertainties originate from physical variability, data uncertainty, and modeling errors in the fatigue crack growth prediction analysis. This study presents an evidential uncertainty quantification (UQ) approach for determining uncertainties involved in revealing the material constants of the metal fatigue crack growth model with imprecise uncertainty information (i.e., epistemic uncertainty). The parameters in fatigue crack growth model are obtained by fitting the available sparse experimental data, and then the uncertainties in these parameters are considered. To alleviate the computational difficulties in the UQ analysis based on evidence theory, an interval optimization method based on differential evolution is used in finding the propagated belief structure. The overall procedure is demonstrated using the results of several replicated experiments on aluminum alloy CCT specimens.

1. Introduction

Uncertainties are prevalent in practical engineering applications and can be categorized as either aleatory uncertainty (also called objective, stochastic, and irreducible uncertainty) due to inherent variability in a physical phenomenon or epistemic uncertainty (also called subjective reducible uncertainty) due to unknown physical phenomena [1]. In the process of fatigue crack growth analysis, the various sources of uncertainty mainly include variability in loading conditions, material parameters, experimental data, and model uncertainty. These uncertainties can affect the analysis for fatigue crack propagation. Numerous experimental studies demonstrated that significant variability in crack propagation occurs even after crack initiation [2,3]. Uncertainty appears at different stages of analysis, and the interaction between these sources of uncertainty cannot be modeled easily. Thus, predicting fatigue behavior due to the various sources of uncertainty is difficult for design engineers or structural analysts.

Numerous uncertain models of crack propagation have been developed to deal with uncertainties observed in large replicate crack propagation tests and thus investigate the uncertainty of crack growth prediction. The quantification for the aleatoric uncertainties is relatively straightforward. Among the existing quantification techniques, Monte Carlo (MC) method is the most frequently used because of its moments than can represent a probability distribution. Karhunen–Loève [4] and polynomial chaos expansions [5] also have the same function. Besterfield et al. [6] combined probabilistic finite element analysis with reliability analysis to predict crack growth in plates. Liu and Mahadevan [7] proposed a concept of equivalent initial flaw size and used MC simulation to predict the probabilistic fatigue life. Jallouf et al. [8] employed probabilistic theory to investigate the reliability of undercut defect in a laser-welded plate made of Ti-6Al-4V titanium alloy. Blacha and Karolczuk [9] validated the effectiveness of the probabilistic model based on the weakest link concept in predicting the fatigue life of steel-welded joints. Fatigue and crack propagation in metals are recognized as stochastic processes [2,3]. Sarkar et al. [10] applied Wiener chaos expansions in estimating fatigue damage in randomly vibrating structures. Beck and Gomes [11] applied polynomial chaos in representing random crack propagation data, in which crack propagation in metals is recognized as a stochastic process. Riahi et al. [12] presented a stochastic collocation method for predicting random crack growth. Zhao et al. [13] combined stochastic collocation approach with Bayesian method in fatigue crack prognosis of metallic material, in which the distributions of random parameters are provided with certain types of distribution, such as normal distribution. Compared with the MC method, this approach is significantly more efficient and time saving and presents more accurate predictions. However, when sufficient data are unavailable or knowledge is lacking, the classical

* Corresponding author.
E-mail addresses: thstj@tongji.edu.cn (H. Tang), lidaweicc123@163.com (D. Li), 772551736@qq.com (J. Li), xue@tongji.edu.cn (S. Xue).

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probability methodology may be inappropriate. Thus, strong statistical information cannot handle uncertainties in a fatigue lifetime prediction problem. In such a case, the usual probabilistic methodologies cannot be used and an alternative approach that can utilize insufficient uncertainty information is required.

Given experimental bounds on the variability of the Paris equation parameters, Worden and Manson [14] investigated the effect of the parameter uncertainty on the estimated lifetime of a cracked metallic plate (Titanium alloy Ti-6Al-4V) using interval arithmetic. Surace and Worden [15] conducted an extended analysis on damage progression within the framework of interval arithmetic. The major problem of the interval approach is that all ranges are entirely independent of one another and the upper and lower bounds are certain. This condition may result in the undesirable overestimation of the true solution set.

In general, the sources of aleatory uncertainty are represented using a probabilistic framework when sufficient data are available. By contrast, epistemic uncertainty cannot be fully characterized by probabilistic approaches because inferring any statistical information may be difficult owing to the lack of knowledge, thereby leading to subjective probabilistic descriptions. Epistemic uncertainty can be represented using various methods, such as interval arithmetic [16], fuzzy sets [17], possibility theory [18,19], information gap decision theory [20], evidence theory [21–24], and imprecise probability [25,26]. Selecting an appropriate mathematical structure to represent epistemic uncertainties can be more challenging than quantifying aleatory uncertainty. For example, the major difficulties in fuzzy set theory lie in that it cannot combine fuzzy sets with probabilistic information and cannot quantify the linguistic uncertainty. The possibility theory has no clear method for combining degrees of belief and probabilistic information. Among these methods, evidence theory has much potential in uncertainty quantification (UQ) and is more general than probability and possibility theories. Evidence theory was first proposed by Dempster [27] and extended by Shafer [21], which offers a framework for naturally modeling epistemic uncertainty and aleatory uncertainty due to its flexibility. It uses plausibility and belief to measure the likelihood of event without the need of additional assumptions. Evidence theory can provide equivalent formulations to convex models, possibility theory, and fuzzy sets, and it can incorporate different types of information in one framework to quantify uncertainty in a system [22]. Recently, some engineering applications with UQ based on evidence theory have achieved significant results [28–33].

Evidence theory has a strong capability to deal with uncertainty modeling and decision under uncertainty when the uncertainty information is imprecise. However, the large computational cost caused by its discrete property severely influences the practicability of evidence theory. To alleviate the computational difficulties in the UQ analysis based on evidence theory, an interval optimization based on differential evolution (DE) for computing bounds method is developed.

In this work, evidence theory is applied in characterizing the uncertainty of a fatigue crack growth model in situations where the uncertainty information is imprecise (i.e., epistemic uncertainty). The available data for the crack growth model material constants are insufficient for assigning a particular probability density function. In such a case, using only one framework (probability theory) to quantify the uncertainty in crack growth prediction may be limited. Thus, evidence theory that is notable for its flexibility and can offer a viable alternative for the purpose of uncertainty propagation is used. Fracture mechanics based on the Paris–Erdogan law [34] is chosen to describe the crack propagation, and initial crack size a_0 and the Paris equation constants C and m are regarded as uncertain variables. The fatigue crack growth data curve fitting analysis of the large replicate experimental results of Virkler et al. [2] and Tian et al. [35] is addressed. The present study aims to investigate the uncertainty of crack propagation using sparse experimental data to explore the feasibility of the approach.

2. Evidential UQ of crack growth model

2.1. Fundamentals of evidence theory

Evidence theory is introduced in this section prior to its application to the uncertainty modeling of crack growth. Evidence theory was originally proposed by Dempster [27] and further developed by Shafer [21] to describe epistemic uncertainty. Among the numerous non-probabilistic methods, evidence theory is the most closely related to probability theory, which is a generalization of classical probability theory. Probability masses can only be assigned to a single event in the UQ with probability theory, and the probability mass function is a mapping R: → [0, 1]. However, in evidence theory, the mass function is not only assigned to a single value but also to sets or ranges. The core of evidence theory is the frame of discernment Θ, which concludes all the possible answers to the investigated problem and all the elements in mutual exclusion between each other. Evidence theory is a mapping from $2^\Theta \to [0, 1]$. Mass function mapping is from $2^\Theta \to [0, 1]$, and A is a subset of 2^Θ, denoted by $A \subseteq 2^\Theta$. This mass function is given by

$$\begin{cases} m(\varnothing) = 0 \\ \sum_{A \subseteq 2^\Theta} m(A) = 1 \end{cases}, \tag{1}$$

where $m(A)$ is also called basic belief assignment (BBA), and it represents confident degree in event A. When $m(A) > 0$, the subset A is called focal element. BBA is estimated by the obtained data or given by experience.

Evidence theory represents uncertainty using a probability interval instead of a probability value. For event A, the lower and upper bounds of uncertainty interval are called the belief function $Bel(A)$ and the plausibility function $Pl(A)$, respectively. $Bel(A)$ represents the confident degree to believe that event A is true, which is the minimum possibility that A occurs, and $Pl(A)$ represents the confident degree to believe that event A is not false, which is the maximum possibility that A occurs. $Bel(A)$ and $Pl(A)$ are given by

$$Bel(A) = \sum_{B \subseteq A} m(B), \tag{2}$$

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \varnothing} m(B). \tag{3}$$

Belief and plausibility functions constitute the lower and upper bounds of proposition A. The interval $[Bel(A), Pl(A)]$ represents the belief degree of proposition A. For information from multiple sources, the combined evidence can be obtained by Dempster's rule [27] of combination. This rule is discussed in detail in [36].

2.2. Evidence-based UQ framework for fatigue crack growth models

Following the brief overview of evidence theory, the evidence-based UQ framework for fatigue crack growth models is presented in this section.

2.2.1. Crack propagation models

The proposed uncertain model of fatigue crack damage is based on a deterministic model of fatigue crack growth [34], which is based on the principle of linear elastic fracture mechanics. The Paris law provides the rate of crack propagation (da/dN) as a function of the amplitude of stress intensity factor (SIF) ΔK: