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Investigation on multiaxial fatigue crack path using polar stress–strain representation

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ABSTRACT

Critical plane concept is widely used as the basis for formulating fatigue damage models. Understanding of fatigue crack behavior, initiation and early growth, under cyclic multiaxial loading is essential for fatigue damage analysis and fatigue life predictions.

Full stress–strain fields surrounding an infinitesimal element at the gauge section were obtained by transforming cyclic hysteresis loops using plane stress–strain transformation relations. These fields were represented on polar diagrams to show the loci of maximum stresses or strains with respect to the plane orientation. Measured crack paths were superimposed onto polar diagrams to understand the crack growth behavior with respect to stress or strain fields. Cracks were found to initiate at both planes of maximum normal or shear strains. It was found that there might be equal chances for a crack to initiate and grow at different paths. Polar diagrams can be used to determine the likelihood regions where crack could possibly initiate and grow.

1. Introduction

Fatigue cracks initiate and grow at certain planes, i.e., persistent slip bands (PSBs). This established fact can be clearly observed in deformed single crystals [1–3]. Fatigue damage is associated with crack formation and critical plane concept originated based on this observation.

The word "critical" was first introduced and used in stress-based approach by Stulen and Cummings [4] and Findley [5]. Later, it was used in the strain-based approach. Brown and Miller [6] suggested two different patterns for crack growth at the surface of a material subjected to multiaxial fatigue loading based on the orientation of the planes of maximum shear strain amplitude with regard to the free surface. They suggested that shear cracks could grow in one of two ways: case A and case B. For case A crack, the shear stress acts on the free surface in a direction parallel to the length of the crack. This represents an in-plane shear stress. No shear stress act perpendicular to the free surface along the crack depth. This type of crack tends to be shallow and have a small aspect ratio as it grows. When case A cracks become longer, i.e. when their dimensions span several grain sizes, stage II crack growth occurs as a result of simultaneous or alternating slip involving more than one slip system [7]. For case B cracks result from out-of-plane shear. Here, the crack initiates at the surface and advances at a 45° angle into the material. The direction of stage II crack growth for case B is also from the free surface into the material. Uniaxial tension fatigue leads to the same shear stress for case A and case B cracks and hence it can facilitate either mode of failure. Torsion, or mixed tension–torsion fatigue loading, however, invariably promotes case A cracks [7,8]. Brown and Miller [6] proposed two separate criteria for each cracking type. However, their definition of the critical plane is only correct for proportional loading when the directions of principal strain are fixed.

Socie [9] categorized fatigue damages depending on the predominant damage mechanism. These are type A in which cracks grow along the maximum shear strain, type B in which crack grows perpendicular to the maximum normal strain direction and type C in which only crack nucleation exists. Socie [9] proposed a different criterion for each damage type. A criterion of maximum shear and normal strain on maximum shear strain plane was proposed for type A damage. On the other hand, a criterion of maximum normal stress and strain on the maximum normal strain plane was proposed for type B damage. For type C, the criterion of the maximum shear and normal stress on maximum shear stress plane was proposed.

Extensive experimental investigations on fatigue cracking behavior of AISI 304 stainless steel, Inconel 718 and normalized SAE 1045 steel were carried out by Socie and co-workers [10–12] but only for axial and torsional loading. The cracking behavior was described by two stages. Stage I refers to shear dominated

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Nomenclature

E	modulus of elasticity
b	normal fatigue strength exponent
b_s	shear fatigue strength exponent
c	normal fatigue ductility exponent
c_s	shear fatigue ductility exponent
γ_a	shear strain amplitude
$\gamma_{\max}$	maximum shear strain
γ'_f	shear fatigue ductility coefficient
γ_φ	shear strain at plane φ
K'	cyclic normal strength coefficient
K'_s	cyclic shear strength coefficient
l_c	crack length
n'	cyclic normal strain hardening exponent
n'_s	cyclic shear strain hardening exponent
N_f	fatigue life
υ	Poisson's ratio
υ_e	elastic Poisson's ratio
υ_p	plastic Poisson's ratio
$\%\,AR$	percentage of area reduction
σ_a	normal stress amplitude
$\sigma_{\max}$	maximum normal stress
σ'_f	normal fatigue strength coefficient
σ_{ut}	ultimate tensile strength
σ_φ	normal stress at plane φ
σ_x	normal stress component in x direction
σ_y	normal stress component in y direction
τ_a	shear stress amplitude
$\tau_{\max}$	maximum shear stress
τ'_f	shear fatigue strength coefficient
τ_φ	shear stress at plane φ
τ_{xy}	shear stress component in xy plane
ε_a	normal strain amplitude
$\varepsilon_{\max}$	maximum normal strain
ε'_f	normal fatigue ductility coefficient
ε_φ	normal strain at plane φ
ε_x^{el}	elastic normal strain component in x direction
ε_x^{pl}	plastic normal strain component in x direction
ε_y	normal strain in y direction
φ	plane orientation
φ_i	increment of plane orientation

growth that is controlled by the microstructure within individual grains. Stage II refers to crack growth that is controlled by the maximum normal stress. For stainless steel, torsional loading was found to result in two cracking behaviors: shear and tension, i.e., Stage I and II. On the other hand, SEM examinations of fracture surfaces on specimens tested under tensile loading showed no evidence of nucleation. Rather, the fracture surfaces appeared to be almost entirely dominated by Stage II growth. Therefore, stainless steel is said to exhibit a mixed cracking behavior. Conversely, Inconel appeared to have a shear dominated cracking in both tensile and torsional loading. This was attributed to the fact that reversed movement of dislocations progressively shears precipitates in localized shear deformation bands that developed during cyclic loading. Therefore, crack propagation occurs along these bands. For the SAE 1045 steel, high density of microcracks was observed at high strain with final failure occurring by a very rapid linking and coalescence of these cracks. This type of damage was termed R system. On the other hand, the S system that dominated the cracking behavior at low strain amplitude exhibited one dominant crack that grew until failure.

Fatigue damage models that are based on the critical plane concept are expected to predict both fatigue life and fatigue cracking plane. There is still no unified definition of fatigue crack initiation. Yet, there is an agreement that there are three classes of fatigue crack growth: microscopic, small and macroscopic [13]. Still these classes are not exactly defined and there is an overlap between them. In general, cracks with lengths less than 10^2 μm are considered microscopic and their growth is governed by the microstructure texture. Based on the general literature, such crack size marks the initiation size. Cracks with lengths between 10^2 and 10^3 μm are considered small and such range represents what is so-called early growth stage. It is widely accepted that fatigue models are valid during initiation and early growth stages. After that, fracture mechanics is used to predict propagation life that is dominated by growth of macroscopic cracks. Experimental observations suggest that materials exhibit different behaviors with respect to crack growth such that major portion of life is consumed in crack initiation and early growth. In other materials crack initiation and early growth represents small percentage not exceeding 10% of the total life. This observation is important for selecting proper fatigue design model.

There are many fatigue models available in the literature that have been verified using rigorous testing scenarios that include mean stress or strain, constant and variable amplitude loading, proportional and non-proportional loading conditions as well as complex loading paths. These models include, but not limited to, Smith–Watson–Topper [9,14], Fatemi–Socie [15] and Jiang [16]. However, Socie et al. [17] conducted a comparative numerical analysis on multiaxial fatigue benchmark experiment performed on simple notched SAE shaft [18]. Five software packages were used to compute the fatigue lives for 75 bending-torsion notched shafts. Socie et al. [17] showed that cumulative probability distribution for in-phase loading test on smooth tubular specimen indicates that there is 99% chance for fatigue life to be predicted within a factor of 2. Conversely, there is 99% chance for fatigue life to be predicted within a factor of 10 when it comes to notched shaft. Socie et al. [17] emphasized on the consideration of complex geometries and loading conditions for evaluating fatigue models.

On the other hand, it is often found that models that are based on completely different critical plane assumptions such as normal or shear still give very similar fatigue life predictions [19–24]. Such observation raises two important questions. First, what is the "critical" plane? The second question is: if two criteria are based on two different critical plane assumptions and predict similar fatigue lives then which one of them is correct?

This investigation aims to study the initiation and early growth behaviors of fatigue crack resulting from multiaxial cyclic loading. Detailed experimental results including multiaxial stress–strain responses and fatigue crack growth records were obtained from the work of Hoffmeyer et al. [25]. No prior definitions of fatigue crack initiation and/or early growth planes were assumed. Rather, full field stress–strain transformation surrounding an infinitesimal element was presented in polar diagrams. Then, observed crack paths, with sizes ranging from 50 to 800 μm, for different loading conditions were superimposed on these polar diagrams. This representation was found to be useful for determining the likelihood regions where crack might possibly initiate and grow.

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