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A non-ordinary state-based peridynamics modeling of fractures in quasi-brittle materials

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ABSTRACT

In this work, we have developed a non-ordinary state-based peridynamics model for brittle fracture in ceramics or fracture in quasi-brittle materials in general. The model is firstly validated by three numerical benchmark tests, and then it is applied to simulate the edge-on impact and drop ball test experiments. We have implemented the modified Johnson Holmquist (JH-2) constitutive damage model into the peridynamics framework at finite strain. Furthermore, the contact algorithm between the projectile and target is discussed. It is shown that the numerical results obtained from peridynamics simulations are in general agreement with those from the experiment. The comparison of experimental and numerical results indicates that the proposed peridynamics model has the ability to capture the damage propagation and other features of the brittle fracture.

1. Introduction

Ceramics exhibit among the highest stiffness and strength of all known material classes dating back to last century. It has been widely used as protective material and military armor date back to the last century, for its good mechanical properties and energy-absorbing ability e.g. [1]. Extensive efforts have been devoted to develop accurate and efficient numerical methods for ceramic armor modeling [2,3]. However, the modeling and predictive simulation of the dynamic response of brittle material has been a challenge for many years. The perforation and penetration in the ballistic application require the computation algorithm having the ability to handle the complex fracture pattern and crack propagation paths, in addition to accurately represent materials behaviors. To date, researchers in the computational mechanics community have developed various techniques to represent the discontinuities in the interpolation of the displacement field, such as the element erosion technique in the finite element method (FEM) [4], the cohesive zone model (CZM), the extended finite element method (X-FEM), and meshfree methods such as Smooth Particle Hydrodynamics (SPH) [5], cracking particles methods (CPM) [6] and Reproducing Kernel Particle Method. In FEM, the element erosion technique will automatically remove highly distorted units from the domain to ensure the stability of the simulation. However, the operation will bring loss to the mass, energy, and the accuracy of the system. In CZM, the cohesive law is typically deployed at the edges or facets of the FEM mesh. Studies have been reported on simulations of the fracture progress based on CZM, e.g. [7,8]. In these studies, the phenomenological elastic or inelastic traction-separation cohesive laws are used to control the fracture behavior. The X-FEM adopts an approach of enrichment of the displacement field, in which discontinuous basis functions are added to standard polynomial finite element basis functions for nodes that belonged to elements that are intersected by a crack to provide so that the discontinuous crack opening displacements can be captured e.g. [9]. The SPH method consolidates a set of discrete particles into an approximation of continuum field by using a kernel integration and interpretation in the current configuration. It, however, may suffer from spurious instabilities and inconsistencies [10]. In CPM the displacement field is decomposed to continuous part and discontinuous part to represent the discontinuity. The crack is treated as a collection of cracked particles. However, instead of enriching the particle and adding additional degrees of freedom, particles are splitted where the cracking criteria are met and introduce crack segments. Also, the methods requires several ingredient of constitutive and cohesive model to determine the cracking criterion

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[11,12]. These approaches requires precise calculation and representation of the crack surfaces, which have been implemented for single- and multiple-cracks problems. However, tracking the crack surfaces can be complex and time-consuming in the ballistic simulation problems. Nevertheless, the interactions between the cracks are somewhat intricate since their evolutions are correlated with each other. The countless possibilities introduced by a huge number of crack surfaces make it a difficult task, to develop a numerical algorithm to deal with these complex geometric evolutions in three-dimension space, including crack initiation and growth, crack surface description, and geometric data bookkeeping.

In recent years, Silling [13] proposed a non-local continuum theory called Peridynamics (PD) theory. In this theory, the displacement field in the reference domain is not required to be differentiable, or even continuous. The theory is a reformulation of continuum mechanics that is better suited for modeling solids with discontinuities, such as cracks. The partial differential equations in the continuum theory are replaced with the integral equations in this method. Peridynamics have two distinct branches, the bond-based peridynamics and the state-based peridynamics. The state-based peridynamics, in particular, have the ability to accommodate any constitutive relations in classic continuum mechanics framework, and is proved to be convergent [14]. State-based peridynamics has been successfully applied to simulate fracture or damage processes. For example, Foster et al. investigated a viscoplastic bar under impact [15]; Mitchell develop a state-based viscoelasticity peridynamics model [16]; Tuniki employed state-based peridynamics to predict the fracture processes of concrete [17]; Lai et al. simulated the fragmentation of geomaterial induced by impluse loads using peridynamics [18]; and Fan et al. performed hybrid peridynamics-SPH simulation of soil fragmentation by blast loads of buried explosive [19,20].

To the best of authors' knowledge, no attempt has been made to develop the state-based PD theory for fracture analysis of quasi-brittle materials, especially, that takes into account the visco-plasticity effects, strain-rate effects, and pressure-dependence. Therefore, the specific aim of this study is to implement a three-dimensional visco-plastic quasi-brittle material model to a state-based PD framework, and to apply it to model brittle fracture. In order to accurately characterize the fracture and damage progress of the brittle material, appropriate constitutive relations are required. To date, various constitutive models for brittle material have been proposed to describe and predict the mechanical responses and behaviors of brittle materials, such as the modified Wilkin's model [21], the Rajendran-Grove Model [22], and the Johnson–Holmquist Model [23]. In this work the modified Johnson Holmquist (JH-2) model is adopted to characterize the plastic behaviors and fracture feature of the ceramic. The JH-2 model has been implemented into many commercial FEM software packages, such as LS-DYNA [24,25] and ABAQUS [26]. This constitutive model has been widely used in numerical simulations of ceramics, glasses and rocks, e.g. [28–30]. The JH-2 ceramic model has several components, and it incorporates a viscoplastic model, a damage model and an Equation of State (EOS). It is fairly accurate in predicting the behavior of ceramics.

In this paper, we first construct a state-based peridynamics formulation for the JH-2 visco-plasticity model for quasi-brittle materials. Since brittle material often encounters large deformation and fragmentation under extremely penetration, the JH-2 model at finite deformation with Rubinstein and Atluri algorithm is incorporated into the constitutive updating. Numerical examples are presented to verify the implementation against analytical solutions. Finally, the JH-2 model within the peridynamics formulation are employed to simulate edge-on impact problems and drop ball test.

This paper is organized into four sections. In Section 2, we first review the mathematical formulations of state-based peridynamics, and then recall the classic Johnson Holmquist constitutive model. After that, the implementation of the nonlinear constitutive update under finite deformation and the related peridynamics formulations are presented and discussed. The key algorithm involved in this work is also introduced. In Section 3, we present numerical examples that are conducted to verify the proposed peridynamics model for brittle materials, which are compared to the analytical solution and relevant experimental results. Finally, a few closing remarks are made in Section 4.

2. Methodology

2.1. Peridynamics formulation

In PD theory, a continuum solid $\mathscr{B}$ is discretized into a set of individual material points $\mathbf{x}_i, i = 1, 2 \ldots \infty$, that carry associated mass density ρ_{x_i}, volume V_i, where i is the index of each particle. Interactions between material points are described directly by the prescribed response function $\mathbf{f}$, which contains all of the constitutive information associated with the material. All the material points $\mathbf{x}_j$ within a certain distance δ with particle $\mathbf{x}_i$ establish interactions with each other. The region $\mathscr{H}$ in which the interactions exist is the *horizon* of particle $\mathbf{x}_i$, typically defined as a circle in 2d or a sphere in 3d with a radius δ. All the neighboring material points $\mathbf{x}_j$ that fall into the horizon of $\mathbf{x}_i$ are called a family of $\mathbf{x}_i$, $\mathscr{H}_{\mathbf{x}_i}$. The relative position vector from particle $\mathbf{x}_i$ to $\mathbf{x}_j$ in the reference configuration is denoted by $\boldsymbol{\xi}_{ij}$ and their relative displacement in current configuration by $\boldsymbol{\eta}_{ij}$

$$\boldsymbol{\xi}_{ij} = \mathbf{x}_j - \mathbf{x}_i, \qquad \boldsymbol{\eta}_{ij} = \mathbf{u}[\mathbf{x}_j, t] - \mathbf{u}[\mathbf{x}_i, t] \tag{1}$$

Then the current relative position between the material points can be represented by $\boldsymbol{\xi}_{ij} + \boldsymbol{\eta}_{ij}$. The governing equations of motion of peridynamics are then expressed as

$$\rho_i \ddot{\mathbf{u}}[\mathbf{x}_i, t] = \int_{\mathscr{H}_{\mathbf{x}_i}} \mathbf{f}(\mathbf{u}[\mathbf{x}_j, t] - \mathbf{u}[\mathbf{x}_i, t], \mathbf{x}_j - \mathbf{x}_i) dV_{\mathbf{x}_j} + \mathbf{b}[\mathbf{x}_i, t] \tag{2}$$

in which, $\mathbf{u}[\mathbf{x}, t]$ is the displacement of the particle $\mathbf{x}_i$ in the solid at time t, while $\mathbf{f}(\mathbf{u}[\mathbf{x}', t] - \mathbf{u}[\mathbf{x}, t], \mathbf{x}' - \mathbf{x})$ is a function defined as the pairwise interaction force that particle $\mathbf{x}'$ exerts on the particle $\mathbf{x}$. $\mathscr{H}$ is the horizon of particle $\mathbf{x}$. $\mathbf{b}[\mathbf{x}, t]$ is an external body force density vector.

Such particle based non-local formulation gives Peridynamics the ability to represent discontinuity naturally and spontaneously. In this work, the non-ordinary state-based Peridynamics (NO-SBPD) theory is utilized to model the complex damage and fracture, while taking strain-rate effects into account. The governing equation of NO-SBPD is given as follows:

$$\rho_i \ddot{\mathbf{u}}[\mathbf{x}_i, t] = \int_{\mathscr{H}_{\mathbf{x}_i}} (\underline{\mathbf{T}}[\mathbf{x}_i, t]\langle\boldsymbol{\xi}_{ij}\rangle - \underline{\mathbf{T}}[\mathbf{x}_j, t]\langle\boldsymbol{\xi}_{ji}\rangle) dV_{\mathbf{x}_j} + \mathbf{b}[\mathbf{x}_i, t] \tag{3}$$

where all the definitions of Eq. (2) hold, and $\underline{\mathbf{T}}[\mathbf{x}_i, t]\langle\boldsymbol{\xi}_{ij}\rangle$ is defined as the force state acted on particle $\mathbf{x}_i$ due to particle $\mathbf{x}_j$. Following [31], the force state can be associated with the first Piola–Kirchhoff stress $\mathbf{P}_i$ using following relationship:

$$\underline{\mathbf{T}}[\mathbf{x}_i, t]\langle\boldsymbol{\xi}_{ij}\rangle = \omega(\boldsymbol{\xi}_{ij}) \mathbf{P}_{x_i} \mathbf{K}_{x_i}^{-1} \langle\boldsymbol{\xi}_{ij}\rangle \tag{4}$$

where $\underline{\omega}(\boldsymbol{\xi}_{ij})$ is the influence function defined on the bond length of each material bond $\boldsymbol{\xi}_{ij}$, i.e. $\xi_{ij} = |\boldsymbol{\xi}_{ij}|$; and $\mathbf{K}_{x_i}$ is the symmetric shape tensor of particle i defined in the reference configuration by

$$\mathbf{K}_{x_i} = \int_{\mathscr{H}_{\mathbf{x}_i}} \omega(\boldsymbol{\xi}_{ij}) \boldsymbol{\xi}_{ij} \otimes \boldsymbol{\xi}_{ij} dV_{\mathbf{x}_j} \tag{5}$$

Based on continuum mechanics, the relation between $\mathbf{P}_{x_i}$ and Cauchy stress σ_{x_i} is written by

$$\mathbf{P}_{x_i} = \mathscr{J} \sigma_{x_i} \mathbf{F}_{x_i}^{-T} \tag{6}$$

where $\mathscr{J} = \det \mathbf{F}_{x_i}$, and $\mathbf{F}_{x_i}$ is the non-local deformation gradient defined at particle i by