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Impact simulation by an Eulerian model for interaction of multiple elastic-plastic solids and fluids

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A B S T R A C T

A multiphase Eulerian formulation for the interaction of visco-plastic compressible solids and compressible fluids is proposed. The plasticity effects in solids are described by relaxation terms in the governing equations which are compatible with the Von Mises yield criterion. The visco-plastic model is validated on experimental data in a range of the impact velocity : a high velocity symmetric rod-on-rod impact experiments, as well as low velocity impacts of jelly-like materials. A good agreement between numerical and experimental results is found.

1. Introduction

Solid-fluid interactions in the case of extreme deformations appear in many industrial applications (blast effects on structures, hypervelocity impacts,...). This kind of problems may involve high pressures and strain rates as well as a high density ratio. The hyperelastic models [1–7] for which the stress tensor is defined in terms of a stored energy function are well adapted to treat accurately such problems. The hyperelastic models are conservative by construction. They are also objective and thermodynamically consistent. In this paper, a multi-component hyperelastic Eulerian formulation is used to compute several impact test cases [8]. The modelling is based on a 'diffuse interfaces method' which was developed for multi-component fluids [9–11] and generalized to the case of interaction of multiple solids and fluids [12,13]. Relaxation terms for the accurate description of plastic deformations proposed in [7] have been added. No hardening parameter is used to deal with the evolution of the yield strength.

The paper is organized as follows. In Section 2, the mathematical model is presented. In Section 3, the numerical method is briefly described. Two test cases are studied in Section 4. In particular, a symmetric copper rod impact is computed and compared to the experimental data provided in [14]. Then, a low velocity clay suspension impact is studied and compared to the experimental results obtained in [15].

2. Viscoplastic model

2.1. Eulerian multi-component formulation of hyperelasticity

Hyperelastic models have been intensively studied in the past few years [1–6,16–20]. In this paper, we consider a modified conservative hyperelastic formulation adapted to the case of isotropic solids. The Eulerian formulation of the multi-component hyperelasticity proposed in [12,13,21] is considered. The numerical algorithm for solving this model is based on the generalization of the discrete equations method developed earlier for multi-component fluids in [9–11] and multi-component solids [8].

As we deal with non-equilibrium flows, each component admits its own equation of state and its own stress tensor. This approach allows us to treat configurations involving several solids and fluids. The discrete equations are obtained by integrating the conservation laws over a multiphase control volume. The general continuous model is written hereafter for the phase k (in 1D case for the sake of simplicity).

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$$
\begin{cases}
\frac{\partial \alpha_k}{\partial t} + u_I \frac{\partial \alpha_k}{\partial x} = 0, \\
\frac{\partial (\alpha\rho)_k}{\partial t} + \frac{\partial (\alpha\rho u)_k}{\partial x} = 0, \\
\frac{\partial (\alpha\rho u)_k}{\partial t} + \frac{\partial (\alpha\rho u^2 - \alpha\sigma_{11})_k}{\partial x} = -\sigma_{11,I} \frac{\partial \alpha_k}{\partial x}, \\
\frac{\partial (\alpha\rho v)_k}{\partial t} + \frac{\partial (\alpha\rho uv)_k}{\partial x} + \frac{\partial (-\alpha\sigma_{12})_k}{\partial x} = -\sigma_{12,I} \frac{\partial \alpha_k}{\partial x}, \\
\frac{\partial (\alpha\rho w)_k}{\partial t} + \frac{\partial (\alpha\rho uw)_k}{\partial x} + \frac{\partial (-\alpha\sigma_{13})_k}{\partial x} = -\sigma_{13,I} \frac{\partial \alpha_k}{\partial x}, \\
\frac{\partial (\alpha\rho E)_k}{\partial t} + \frac{\partial (\alpha\rho E u - \alpha\sigma_{11} u - \alpha\sigma_{12} v - \alpha\sigma_{13} w)_k}{\partial x} \\
\qquad = -(\sigma_{11,I} u_I + \sigma_{12,I} v_I + \sigma_{13,I} w_I) \frac{\partial \alpha_k}{\partial x}, \\
\frac{\partial (\alpha a^\beta)_k}{\partial t} + \frac{\partial (\alpha a^\beta u)_k}{\partial x} + (\alpha b^\beta)_k \frac{\partial v_k}{\partial x} + (\alpha c^\beta)_k \frac{\partial w_k}{\partial x} = 0, \quad \beta = 1,2,3 \\
\frac{\partial b_k^\beta}{\partial t} + u_k \frac{\partial b_k^\beta}{\partial x} = 0, \quad \beta = 1,2,3 \\
\frac{\partial c_k^\beta}{\partial t} + u_k \frac{\partial c_k^\beta}{\partial x} = 0, \quad \beta = 1,2,3.
\end{cases}
\tag{1}
$$

Here, for k^{th} phase: α_k is the volume fraction, ρ_k is the phase density, $\mathbf{u}_k = (u_k, v_k, w_k)^T$ is the velocity field, $\boldsymbol{\sigma}_k$ is the stress tensor:

$$\boldsymbol{\sigma}_k = \mathbf{S}_k - p_k \mathbb{I}, \tag{2}$$

where $\mathbf{S}_k$ is the deviatoric part of the stress tensor and p_k is the thermodynamical pressure. As the model belongs to the class of hyperelastic models, the stress tensor can be expressed as the variation of the internal energy (exact expressions are given in the following subsection). The evolution equations of hyperelasticity are written for deformation measures (in particular, for the Finger tensor defined below). E_k is the total energy associated to the phase k and is given by the following expression:

$$E_k = \frac{\| \mathbf{u}_k \|^2}{2} + e_k(\eta_k, \mathbf{G}_k), \tag{3}$$

where η_k is the entropy of the phase k and $\mathbf{G}_k$ is the Finger tensor. The exact expressions of $e_k(\eta_k, \mathbf{G}_k)$ in (3) will be given in the next subsection.

The variables with subscripts 'I' are the 'interface' variables. They are obtained directly when solving the Riemann problem. Some details about the 'interface' variables are given in the next section. The model is thermodynamically consistent and satisfies the second principle of thermodynamics. The proof is not straightforward. Nevertheless, the thermodynamic consistency has been verified on numerical test cases (for example, a shock wave propagation through a medium in presence of material interfaces). In the right hand side of the system (1), non conservative terms are present: these terms exist if the volume fraction gradient is non zero.

The geometric variables $a_k^\beta, b_k^\beta, c_k^\beta$ related to the deformation gradient will now be defined. To simplify the presentation, we will not further use in this section the subscript k for unknowns.

Let us define the Finger tensor $\mathbf{G}$ as the inverse of the left Cauchy-Green tensor $\mathbf{B}$: $\mathbf{G} = \mathbf{B}^{-1}$. The Finger tensor can also be expressed in the form:

$$
\mathbf{G} = \sum_{\beta=1}^{3} \mathbf{e}^\beta \otimes \mathbf{e}^\beta, \quad \mathbf{e}^\beta = (a^\beta, b^\beta, c^\beta)^T, \quad \mathbf{e}^\beta = \nabla X^\beta, \quad \beta = 1,2,3,
$$
$$
\mathbf{F}^{-T} = (\mathbf{e}^1, \mathbf{e}^2, \mathbf{e}^3).
$$

Here X^β are the Lagrangian coordinates, and $\mathbf{F}$ is the deformation gradient. The gradient is taken with respect to the Eulerian coordinates. In the next subsection the equation of state is presented, allowing the system closure. Different relaxation phenomena can easily be added into the model (pressure and velocity relaxation, phase transitions ...).

2.2. System closure

The closure of the system is performed by using an equation of state presented in a separable form [22]:

$$e(\eta, \mathbf{G}) = e^h(\rho, \eta) + e^e(\mathbf{g}), \; \mathbf{g} = \frac{\mathbf{G}}{|\mathbf{G}|^{1/3}}, \tag{4}$$

where $|\mathbf{G}|$ denotes the determinant of the tensor $\mathbf{G}$. This formulation has been used in particular in [12,21,23,24]. With such a formulation, the pressure is determined only by the hydrodynamic part of internal specific energy $e^h(\rho, \eta)$. The deviatoric part of the stress tensor can be expressed using the shear part of the specific internal energy $e^e(\mathbf{g})$. The hydrodynamic part of the energy satisfies the Gibbs identity:

$$\theta d\eta = de^h + p d\tau,$$

where τ is the specific volume ($\tau = 1/\rho$) and θ is the temperature. The expression of the deviatoric part of the stress tensor S is:

$$S = -2\rho \frac{\partial e^e}{\partial \mathbf{G}} \mathbf{G}.$$

The hydrodynamic part of the internal specific energy is taken as the stiffened gas equation of state:

$$e^h(\rho, p) = \frac{p + \gamma p_\infty}{\rho(\gamma - 1)}. \tag{5}$$

In [25], a family of rank-one convex stored energies for isotropic compressible solids with a single parameter (denoted by $\tilde{a}$) is proposed:

$$e^e(\mathbf{G}) = \frac{\mu}{4\rho_0}\left(\frac{1-2\tilde{a}}{3} j_1^2 + \tilde{a} j_2 + 3(\tilde{a} - 1)\right), \qquad j_m = tr(\mathbf{g}^m), m = 1,2. \tag{6}$$

Here, μ is the shear modulus of the considered material and ρ_0 is the reference density. Using the criterion proposed in [25,26], it has been proven that with the equations of state (5) and (6), the equations are hyperbolic for any $\tilde{a}$ such that $-1 \le \tilde{a} \le 0.5$. The relation (6) involves the following expression for the deviatoric part:

$$\mathbf{S} = -\mu \frac{\rho}{\rho_0}\left(\frac{1-2\tilde{a}}{3} j_1 \left\{ \mathbf{g} - \frac{j_1}{3}\mathbb{I} \right\} + \tilde{a}\left\{ \mathbf{g}^2 - \frac{j_2}{3}\mathbb{I} \right\}\right). \tag{7}$$

One can notice that for the value $\tilde{a} = -1$, the equation of state describes neoHookean solids. Its expression is the following:

$$e^e(\mathbf{G}) = \frac{\mu}{4\rho_0}(j_1^2 - j_2 - 6). \tag{8}$$

The energy (8) is, in particular, suitable for the description of jelly-type materials. In the case of metals, the value $\tilde{a} = 0.5$ can be chosen, where the equation of state becomes:

$$e^e(\mathbf{G}) = \frac{\mu}{8\rho_0}(j_2 - 3). \tag{9}$$

2.3. Viscoplasticity modelling

An important class of hyperbolic models describing the plastic behavior of materials under large stresses has been proposed, for example in [1,3,16,17]. An extension of this approach has been proposed in [7,13,21,27] to include material yield criteria (Von Mises). The relaxation terms are constructed in such a way that they are compatible with the mass conservation law and consistent with the second law of thermodynamics. The Von Mises yield limit is reached at the end of the relaxation step. The built model belongs to Maxwell type model, where the intensity of the shear stress decreases during the relaxation.