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Shape sensitivity analysis and shape optimization in planar elasticity using the element-free Galerkin method ☆

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Abstract

This paper demonstrates that the element-free Galerkin (EFG) method can be successfully used in shape design sensitivity analysis and shape optimization for problems in 2D elasticity. The continuum-based variational equations for displacement sensitivities are derived and are subsequently discretized. This approach allows one to avoid differentiating the EFG shape functions. The present formulation, that employs a penalty method for imposing the essential boundary conditions, can be easily extended to 3D and/or non-linear problems. Numerical examples are presented to show the capabilities of the current approach for calculating sensitivities. The flexibility of the EFG method, that eliminates the element connectivity requirement of the finite element method (FEM), permits solving shape optimization problems without re-meshing. The problem of shape optimization of a fillet is used to demonstrate this fact. Smoother stresses and better accuracy for points close to the boundary allow for a better EFG solution compared to published results using the FEM, the boundary element method (BEM) or the boundary contour method (BCM). Furthermore, for the EFG approach, grid-optimization appears unnecessary.

Keywords: Meshless methods; Element-free Galerkin (EFG); Shape sensitivity analysis; Shape optimization; Linear elasticity

1. Introduction

The regular finite element method (FEM) is marked by some shortcomings such as: discontinuous secondary variables that require a costly and not always satisfactory smoothing procedure, need of re-meshing in case of severe distortion of the mesh, locking for nearly incompressible materials, inaccurate results near the boundary of a domain, etc. These shortcomings are even more acute when dealing with shape design optimization problems. In this type of problems, it is essential to obtain accurate primary and secondary variables at or near the boundary of the domain since the boundary usually changes during an iterative optimization process. Also, in an FEM-based iterative process for improving the design, the need of re-meshing to avoid loss of accuracy due to distortion often becomes a necessity and represents a burden in terms of computational effort. Furthermore, interpolation of variables across meshes can cause significant errors.

To overcome some of these difficulties, the element-free Galerkin (EFG) method was proposed by Belytschko et al. [6] in the realm of solid mechanics. The EFG method is a Galerkin discretization scheme based on the moving least-squares (MLS) approximation (see e.g., [18]). The precursor of the EFG method

☆ Florin Bobaru would like to dedicate this paper to the memory of Professor Iulian Beju.

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was the diffuse element method (DEM) proposed by Nayroles et al. [22]. The EFG method presented in [6] includes certain terms in the derivatives of the MLS approximants that are omitted in the DEM, uses a Lagrange multiplier to enforce Dirichlet boundary conditions and employs a large number of quadrature points arranged in a cell structure over the domain.

Many problems in solid mechanics, ranging from elasticity to dynamic fracture problems, have been solved using the EFG method. The review paper by Belytschko et al. [5] cites many results from the EFG method, and also gives an account of other related methods like the *h–p*-clouds of Duarte and Oden [11], reproducing Kernel particle methods (RKPM) of Liu et al. [19], etc. The EFG method provides higher rates of convergence than the FEM, as well as smooth secondary variables at no additional cost. Moreover, locking is avoided and re-meshing is eliminated since the mesh (cells) used in the EFG is only for integration purposes. Also, there is no loss of accuracy near the boundary of the domain under consideration. Another feature of the EFG, with significant impact on shape sensitivity analysis, is that the solution is rather insensitive with respect to the arrangement of the nodes (see e.g., [6]). Therefore, an optimization procedure for the position of nodes, for some fixed number of nodes, seems unnecessary. For all these reasons, the EFG method seems to constitute a very appealing approach for use in sensitivity analysis and shape optimization.

Design sensitivity analysis (DSA) is concerned with finding the variation (derivative) of a response measure due to a variation of some design parameters. In shape sensitivity analysis, the design parameters describe the geometry of the domain. The sensitivities are needed in a gradient-based optimization process in order to provide the gradients of the objective function (that could be, for example, the area or the volume of a structural part) and of the constraints (which could be, for example, constraints on the admissible stress in the body). Two review papers in these areas are Haftka and Grandhi [13] and Tortorelli and Michaleris [29]. In [13], a series of difficulties are presented related to the application of the FEM or the boundary element method (BEM) for solving shape optimization problems.

For shape design sensitivity calculations, two formulations are possible: the material derivative and the control volume approach (see e.g., [3]). Within each of these formulations, two methods can be applied: the direct differentiation method (DDM) (see e.g., [3,15,35]), and the adjoint method (AM) (see e.g., [2,15,30]). From a computational point of view, the DDM is more efficient in the case when the number of design variables is lower than the number of constraints, whereas the AD is a better choice when the reverse is true (see e.g., [15]). A recent paper by Grindeanu et al. [12] addresses the DSA for hyperelastic structures using the RKPM.

In this paper, we develop, for the first time, shape design sensitivity analysis in the EFG method context. We employ a continuous formulation using the material derivative approach. The DDM is applied since in the shape optimization example we show it is more advantageous to do that. The present formulation is used to derive displacement, stress and strain sensitivities that can be used in shape optimization. The material derivative of the weak form is obtained prior to discretization. This, together with a particular approximation for the sensitivities of the displacements, enables us to avoid differentiating the EFG shape functions with respect to the design variables.

Several examples, for which analytical solutions are available, are provided to test the accuracy of this development. Displacement sensitivities are also tested against a finite difference method (FDM) solution for the case of a fillet. Subsequently, shape optimization of the fillet is performed and this clearly demonstrates the ability and versatility of the EFG method for this kind of problems where classical finite element and BEMs have to surmount significant difficulties.

This paper is organized as follows: In Section 2, we briefly describe the EFG method with a penalty formulation for imposing the essential boundary conditions. In Section 3, we formulate the DSA problem in the context of the EFG method using the DDM. Section 4 presents two numerical examples for testing the accuracy of displacement and stress sensitivities against known analytical solutions. In Section 5, we test displacement sensitivities for a fillet, obtained via the DDM, by comparing them with their finite differences counterparts. Shape optimization of the fillet is then performed with the EFG using, for convenience, FDM derived sensitivities. Comparisons with published results using the FEM, BEM or the boundary contour method (BCM) (see [21]) are made. The paper ends with some conclusions that are presented in Section 6.