International Journal of Impact Engineering 108 (2017) 47–62

Contents lists available at ScienceDirect

International Journal of Impact Engineering

journal homepage: www.elsevier.com/locate/ijimpeng

Constitutive equations for porous solids with matrix behaviour dependent on the second and third stress invariants

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ARTICLE INFO

Article History:
Received 5 February 2017
Revised 2 May 2017
Accepted 3 May 2017
Available online 9 May 2017

Keywords:
Porous materials
Effective yield criterion
Lode angle

ABSTRACT

Constitutive equations are developed for voided materials and ductile fracture taking into account possible effects of Lode angle in the yielding behaviour of the matrix. The Gurson criterion (Gurson, 1977) [4] is generalized to such circumstances. A semi-closed form expression , similar to the Gurson criterion is obtained for the effective yield criterion for the porous solid and involves four different functions , all dependent on the macroscopic stress triaxiality and Lode angle but are not generally available in closed form. In parallel, a parametric representation of the effective yield criterion is provided which allows for the derivation of closed form results for pure shear stress states and also at very high stress triaxialities. In the former case corresponding to a zero macroscopic mean stress, the contour of the yield domain in the π-plane has exactly the shape of the yield surface of the matrix in the deviatoric plane but a size reduced by a factor $1-f$, with f the porosity of the voided material. In the latter, effective yield stresses for the voided material are slightly different from the Gurson result and found to be set by the yield stress at a microscopic stress Lode angle $\frac{\pi}{3}$ for very high positive triaxiality and by the yield stress at a microscopic stress Lode angle 0 for very high negative triaxiality. Various numerical results are furnished to illustrate all the obtained results.

1. Introduction

An important issue in material mechanics is the development and validation of accurate *macroscopic* constitutive equations for engineering materials allowing good design and in service predictions for structural components and in particular for impact loadings. This is a difficult task and very often the needed constitutive equations are rather obtained in an ad hoc, sometimes empirical and in many cases in a phenomenological way. In these ad-hoc and phenomenologically developed constitutive equations, the microstructure and physical mechanisms responsible for the behaviour and fracture of these materials are usually not taken into account. Translation of this microstructure and mechanisms information to the macroscopic level can be done through homogenization and scale transitions in the spirit of McClintock [1], Rice and Tracey [3] approaches for void growth and fracture and by Gurson [4] for yielding of porous materials. The three contributions are so important that they are still currently in use today. All three contributions were undertaken with a yielding of the matrix obeying a plastic behaviour governed by the von Mises yield criterion. Traditionally, the von Mises criterion is the most utilised criterion because of its mathematically simple form.

There are situations where the von Mises matrix behaviour seems insufficient for reproducing the experimental observations. For instance, Ohashi and Tokuda [5] obtained detailed information about the plastic behaviour of real materials by precise measurement of plastic deformation of thin-walled tubular specimens of initially-isotropic mild steel under combined loading of torsion and axial force. They used trajectories consisting of two straight lines at a constant rate of the effective strain. From these experimental results, they found that the effect of the third invariant of the strain tensor appeared even for proportional deformation consisting of torsion and axial force. Further, they observed the effective stress to drop suddenly with increasing effective strain and that coaxiality between the stress deviator and the plastic strain increment tensor to be seriously disturbed just after the corner of the strain trajectory. These local disturbances are recovered along the second branch of the trajectory. In another important experimental investigation, Rousset [6] measured precisely subsequent yield surfaces for an 2024 aluminium alloy and observed that even in the simpler case of proportional loadings, subsequent yield surfaces are distorded with a corner forming in the loading direction and a flattening in the opposite one. Also, with the use of polycrystal theory of plasticity it was found (see e.g. Hershey [15] and Hosford [16]) that the yield surfaces for fcc-metals do not have the elliptical form described by the von Mises criterion. In another context, the forming limit diagrams are seen to be significantly dependent on the yield surface [9]. Many

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http://dx.doi.org/10.1016/j.ijimpeng.2017.05.004
0734-743X/

aluminium alloys for instance exhibit significant anisotropy in strength, plastic flow and ductility. The use of Hill's original anisotropic criterion [10] (based on the von Mises criterion) has been shown unsuitable for f.c.c metals or for materials exhibiting low r values (where r is the ratio of the width to the thickness strain under uniaxial tension). Experimental evidence shows here that the biaxial flow stress in aluminum alloys is larger than the uniaxial flow stress [7,8] whereas Hill's theory [10] predicts the contrary. A number of theoretical, experimental and numerical investigations exist in the literature with the objective of a better description of yielding of isotropic and anisotropic materials. Thus, for isotropic f.c.c, materials, Hershey [15] and Hosford [16] have proposed the same equation for the description of the yield surface and this will be used in the sequel of this paper. This equation has also been generalized by Logan and Hosford [13] for anisotropic materials. Another criterion was proposed by Hill [11] while other improved yield criteria have been proposed by Barlat et al. [12]. Lademo [17] investigated several of these yield criteria and clearly demonstrated the need of a more complex yield behaviour for aluminium alloys. He also shows that the contours of shear stress change their shape for increasing values of shear stress and this calls for a coupling between the shear and normal stress components in the equation for the yield criterion. To close this paragraph, one can conclude that independently from all the criteria sketched above, and for incompressible plasticity, all involves in a way or another the third stress (Lode angle) invariant even in the isotropic case. This is the subject of this paper aimed at deriving macroscopic constitutive equations for voided materials the matrix of which has a yielding behaviour dependent on both the second and third stress invariants. The general yield function considered herein encompasses most of the usual criteria. The derivation of the effective yield criterion for the porous solid is carried out in the framework of the Gurson approach and the analysis will be limited here to isotropic materials.

Effects of the Lode angle in the Gurson approach appear at two different levels. Beside the fact that the matrix behaviour is dependent on the Lode angle, it also enters in the homogenization process as the stress state in the representative volume cell (a hollow sphere) is heterogeneous. In a recent paper the author and co-workers [24] assessed the effects of the third stress invariant in the yielding of ductile porous solids arising from the later effect by considering a von Mises yielding behaviour for the matrix. This was done by simply avoiding the approximation used by Gurson [4] and considering the full expression of the microscopic dissipation. For small porosities encountered on ductile fracture of metals, observed changes and roles of the Lode angle are found rather small although from the qualitative point of view, non-symmetry of the yield locus and changes on its curvature are observed. However, some changes were found in the intermediate regime of triaxialities and a careful inspection of these changes are seen to be second order effects (of the triaxiality) rather than direct effects of the Lode angle in the yielding of porous materials which only arise at third order of Gurson μ parameter. The coming analysis will consider all these aspects.

Other situations calling for more complex behaviour (either plasticity or fracture) are a number of experimental observations on failure under low or negative triaxialities (McClintock [2], Johnson and Cook [18], Bao and Wierzbicki [14], Barsoum and Faleskog [19] and Fourmeau et al. [23]) . Shear-dominated stress states such as plugging failure in projectile penetration are other examples [20] and many others can be found in the above references. Nahshon and Hutchinson [27] amended for instance the Gurson model in a phenomenological way by making the evolution of the porosity also dependent on the third invariant of the stress. A number of other experimental, theoretical and numerical studies have emerged since on the comprehension and the modelling of ductile fracture at low triaxialities and in particular on the introduction of the third invariant of stress in constitutive equations. The Lode angle effects have been also included in [31] and [32] and studied by Danas and Ponte Castaneda [21,22] in an alternative approach to limit analysis of unit cell and based on second order variational homogenization techniques.

The outline of the paper is as follows. In Section 2, we set the notations used throughout the paper. The constitutive equations of the matrix that we have in mind are described in details in Section 3. Section 4 describes the derivation of the parametric representation of the effective yield surface of the voided material when the Gurson trial velocity field is used. This parametric form is used in Section 5 for various numerical simulations and to obtain some closed form results for hydrostatic and pure loadings. In Section 6 we give a mathematical semi-explicit expression for the equation of the yield domain fully including effects of the Lode angle. Throughout the paper, the results are illustrated using two different yielding behaviour for the matrix.

2. Notations

The paper is concerned with the effective behaviour of porous ductile materials described by a representative volume element V containing voids and the rest occupied by a matrix the constitutive behaviour of which is considered here as incompressible, *isotropic* and rigid-plastic. In all the paper, $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\epsilon}}$ denote the microscopic stress and strain rate in the matrix while the macroscopic stress and strain rate are called respectively Σ and $\dot{\mathbf{E}}$. The latter are defined by

$$\Sigma = <\boldsymbol{\sigma}>_V = \frac{1}{V}\int_V \boldsymbol{\sigma}\, \mathrm{d}V \tag{1}$$

$$\dot{\mathbf{E}} = <\dot{\boldsymbol{\epsilon}}>_V = \frac{1}{V}\int_V \dot{\boldsymbol{\epsilon}}\, \mathrm{d}V \tag{2}$$

where the operator $<\cdot>_V$ refers to averaging over the volume V of the representative volume element.

The invariants of the microscopic stress tensor are the mean stress σ_m, the von Mises equivalent stress σ_{eq} and the stress Lode angle ω defined respectively by

$$\sigma_m = \frac{1}{3}\sigma_{ii}, \quad \sigma_{eq} = \sqrt{\frac{3}{2}s_{ij}s_{ij}} \ \text{ and } \ \omega = \frac{1}{3}\arccos\left(\frac{27}{2}\frac{\det \mathbf{s}}{\sigma_{eq}^3}\right) \tag{3}$$

where $\mathbf{s}$ is the microscopic stress deviator and repeated summation is used. The same invariants will be used for the macroscopic stress $\boldsymbol{\Sigma}$ and are given by

$$\Sigma_m = \frac{1}{3}\Sigma_{kk}, \quad \Sigma_{eq} = \sqrt{\frac{3}{2}S_{ij}S_{ij}} \ \text{ and } \ \Theta = \frac{1}{3}\arccos\left(\frac{27}{2}\frac{\det \mathbf{S}}{\Sigma_{eq}^3}\right) \tag{4}$$

$\mathbf{S}$ is the stress deviator and Θ the Lode angle of the macroscopic stress. For isotropic materials considered here, the investigation range of both ω and Θ can be limited $0 \le \Theta \le \frac{\pi}{3}$. The ordered macroscopic principal stresses are denoted by $\Sigma_1 \ge \Sigma_2 \ge \Sigma_3$. Beside the Lode angle Θ, other equivalent measures can be used to describe effects of the third stress invariant of the macroscopic stress, such as the Lode parameter given by

$$L = \frac{2\Sigma_2 - \Sigma_1 - \Sigma_3}{\Sigma_1 - \Sigma_3} = 3\frac{\Sigma_2 - \Sigma_m}{\Sigma_1 - \Sigma_3} = \sqrt{3}\tan\left(\Theta - \frac{\pi}{6}\right) \tag{5}$$

taking values in the range $-1 \le L \le 1$.

Fig. 1(a) shows the effective yield domain obtained by Gurson in the principal stress coordinate system (Σ_1, Σ_2, Σ_3). The Lode angle Θ is best represented in the octahedral plane (see Lubliner [26]) where Θ is (taken here) as the angle between the projection $\Sigma_1^{'}$ of the maximum principal direction on the octahedral plane and the stress deviator component $\Sigma^{'}$on this plane depicted in Fig. 1(b) showing a section of the yield surface. It is usually convenient to