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A 3D discontinuous Galerkin finite element method with the bond-based peridynamics model for dynamic brittle failure analysis

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ABSTRACT

Peridynamics is a new nonlocal theory that provides the ability to represent displacement discontinuities in a continuum body without explicitly modeling the crack surface. In this paper, an explicit dynamics implementation of the bond-based peridynamics formulation is presented to simulate the dynamic fracture process in 3D elastic solid. Based on the variational theory, the discontinuous Galerkin (DG) approach is utilized to formulate the classic peridynamics governing equation. As a result, the spatial integration can be carried out through finite element approach to enforce the boundary conditions, constraints, contacts as well as to handle the non-uniform mesh in the engineering practices. The classic material parameters, such as the elastic modulus and fracture energy release rate are employed for the determination of material response and failure in brittle material. Several numerical benchmarks are conducted to invest the convergence and mesh sensitivity of simulations of dynamic crack propagation process with different refinements. The results demonstrate that the proposed peridynamics formulation can capture the 3D dynamic crack process in brittle material effectively and accurately including multi-crack nucleation, propagation and branching.

1. Introduction

The numerical simulation of material failure is a longstanding challenge in the computational mechanics society as well as in the industry. The main difficulty arises from the incompatibility between the physical discontinuities emerging from material failure and the partial differential equations utilized by the classic continuum mechanics theory to describe the material response of a solid body. Another numerical difficulty of the material failure simulation in solids is the challenge of maintaining an adequate data structure representing the evolving crack surfaces during failure process. Much effort has been devoted to overcome these numerical challenges. At the continuum level, the XFEM method [1] introduces the level set method into the finite elements and implicitly determines the position and orientation of crack tips. However, as a mesh-based method, a sophisticated book-keeping algorithm is required to track the crack surface which turns out to be very difficult in 3D problems. The Cohesive FEM [2] method can naturally represent the evolving discontinuity in computational domain. However, cohesive laws in the cohesive model are phenomenological which not only ruins the consistency of the material property but also leads to a convergence problem even in an isotropic solid. The meshfree methods [3] also have been developed to model the material failure. Compared to the XFEM, the meshfree methods update the connectivity with customized approximations and represent the moving boundary conditions with less effort. However, the current techniques to handle multiple cracks seem to not be robust and require further research. On the other hand to avoid the localization issue and thus the mesh sensitivity issue, the nonlocal theories which have been developed since the late of 1970s [4] are used in XFEM and meshfree methods. To evade the spatial differential operation nearby crack surface, the continuum weakly or strictly nonlocal models [5] have been developed. These models lead to a formulation where the spatial derivatives in the weak form of corresponding governing equations are smeared along the material failure surfaces.

Peridynamics is one of the nonlocal methods proposed by Silling [6,7]. It has been considered as a viable and efficient numerical method for the material and structural failure problems. Peridynamics theory replaces the spatial differential term in the classical mechanical theories by a nonlocal integral term that assembles the interaction forces of a material point with its neighbors. The first peridynamics model was presented in 1997. It was named bond-based peridynamics and was applied to the brittle materials [6]. In the bond-based peridynamics model, each material point interacts with its neighbors in a compact zone. The interaction between two material points is called a "bond" which is independent with each other. The pairwise bond forces are collinear with the line of a bond and have

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opposite directions. The bond-based peridynamics model is well-developed and has been applied to the simulation of damage and fracture in the brittle materials [8], the reinforced concrete materials [9], the composite laminate structures [10], the brazed joints [11] and geo-materials [12]. The three-dimensional formulas of the bond-based peridynamics can be derived from a pair-wise elastic potential model which shall result in a constant Poisson's ratio $\nu = 0.25$ rooted in the so-called Cauchy relation, i.e., the elastic modulus tensor satisfies the relations as $D_{1122} = D_{1212}$. To address this restriction, the so-called ordinary and non-ordinary state-based peridynamics models [13,14] were proposed in which the bond forces are dependent each other in contrast with that in the bond-based peridynamics. The state-based peridynamics evaluates the bond force based on the multi-body potential function which has the capability to represent both the effect of volume and shear. Although the state-based peridynamics model has the potential to solve the general material failure problems, it is still an ongoing research topic. There are some technical issues [15,16] that remain to be further discussed.

The motivation of the peridynamics theory is the prediction of material damage in a 3D solid. Thus the peridynamics computational space is firstly partitioned by a set of material particles. Subsequently, the nonlocal integral term of peridynamics theory is implemented by the nodal integral approach [17]. This meshfree type of implementation can capture the crack path freely. However, the boundary condition enforcement cannot follow the standard way of the meshfree Galerkin formulations. Another shortcoming is that the accuracy of computation decays dramatically in the case of non-uniform discretization. An alternative way to perform the spatial integration and avoid those numerical defects in peridynamics models is constructing an approximation field of the kinematic quantity by finite element (FE) shape function [18,19]. Based on this argument, the integration operation can be carried out through Gauss integration points. Several studies [20,21] have been conducted to verify that the peridynamics model can be implemented in the FEM framework with nonlocal boundary conditions. The FEM peridynamics approach inherits the advantages of FEM method such as the straightforward boundary condition enforcement and the robustness in non-uniform discretization. To represent the strong discontinuities in FEM peridynamics, the continuous approximation field is replaced by a piecewise continuous field which results in a discontinuous Galerkin formula for peridynamics [19,22,23] in quasi-static analysis. The piece-wise continuous approximation implies the capability to represent the crack surfaces automatically. The research reports [22,23] indicate that this model can lead to a stable solution for the quasi-static problems.

Instead of modeling brittle fracture as a quasi-static problem, this paper presents the explicit dynamics bond-based peridynamics formulas using the FEM discontinuous Galerkin theory for the dynamic fracture problems. A distributive micro modulus of each bond is calculated from the classic elastic modulus to enforce an equivalent elastic energy density from the peridynamics and continuum mechanics theory. This paper is organized as follows. In section 2, the basic bond-based peridynamics formulations are reviewed. Section 3 constructs the 3D dynamic discontinuous Galerkin weak form of the bond-based peridynamics model. The relations between the nonlocal peridynamics quantities and classic mechanics quantities are derived based on the equivalent elastic energy density in Section 4. After that, several benchmark problems are presented in Section 5. Final remarks are given in Section 6.

2. The bond-based peridynamics model

The bond-based peridynamics model can be considered a macro-scale molecular dynamics model. The dynamic motion of a peridynamics point is governed by the collective of the interaction forces of this point and its neighboring points in a compact zone. The equation of motion of any point ($\boldsymbol{X}$) at reference configuration at time t is:

$$\rho\ddot{\boldsymbol{u}} = \sum_{H_X} \boldsymbol{f}(\boldsymbol{u}(\boldsymbol{X}',t) - \boldsymbol{u}(\boldsymbol{X},t), \boldsymbol{\xi}) dV_{\boldsymbol{X}'} + \boldsymbol{b}(\boldsymbol{X},t) \tag{1}$$

where H_X is a compact neighborhood zone of $\boldsymbol{X}$, named as horizon. The horizon of $\boldsymbol{X}$ is defined as $H_X = \{\boldsymbol{X}' \| \boldsymbol{X}' - \boldsymbol{X}| \le \delta\}$, where δ is the radius of a sphere centered at $\boldsymbol{X}$. $\boldsymbol{\xi}$ denotes a bond as $\boldsymbol{\xi} = \boldsymbol{X}' - \boldsymbol{X}$. The pair of interaction forces ($\boldsymbol{f}$) between $\boldsymbol{X}$ and $\boldsymbol{X}'$ is collinear with the bond and has opposite orientation which is determined by the relative displacement of two points: $\boldsymbol{\eta} = \boldsymbol{u}(\boldsymbol{X}',t) - \boldsymbol{u}(\boldsymbol{X},t)$. $\boldsymbol{b}$ is the prescribed body force density. The integration term in Eq. (1) collects all the bond forces imposed to $\boldsymbol{X}$.

There are two important hypotheses of the bond force: (1) the bond force is a short range force, i.e., it only appears inside the compact zone:

$$\boldsymbol{f}(\boldsymbol{\eta}, \boldsymbol{\xi}) = 0 \quad when\, |\boldsymbol{\xi}| > \delta. \tag{2}$$

(2) $\boldsymbol{f}$ is a pairwise interaction force satisfying:

$$\boldsymbol{f}(-\boldsymbol{\eta}, -\boldsymbol{\xi}) = -\boldsymbol{f}(\boldsymbol{\eta}, \boldsymbol{\xi}), \tag{3}$$

which ensures the conservation of linear momentum [14] (Fig. 1).

In the bond-based peridynamics model, the material is considered as *microelastic* implying that a bond force is related to a *micropotential* w:

$$\boldsymbol{f}(\boldsymbol{\eta}, \boldsymbol{\xi}) = \partial w(\boldsymbol{\eta}, \boldsymbol{\xi}) \big/ \partial \boldsymbol{\eta} \tag{4}$$

where the *micropotential* is a peridynamics concept which is a measurement of the elastic energy stored in a bond. In contrast to that of classic continuous mechanics, the *micropotential* has the unit of N/m^5. Consequently, the bond force $\boldsymbol{f}(\boldsymbol{\eta}, \boldsymbol{\xi})$ has the dimension of N/m^6. The energy density of $\boldsymbol{X}$ can be collected through all its bonds:

$$W = \frac{1}{2} \int_{H_X} w(\boldsymbol{\eta}, \boldsymbol{\xi}) dV_{\boldsymbol{X}'}. \tag{5}$$

The governing equation of the bond-based peridynamics is constructed from the derivative of energy equation. Here the formations of $w(\boldsymbol{\eta}, \boldsymbol{\xi})$ represent the material types. It can be linear, non-linear isotropic or anisotropic materials. This paper employs the *prototype microelastic brittle* (PMB) material model [15]. The PMB model is a linear isotropic material model in which each bond is considered

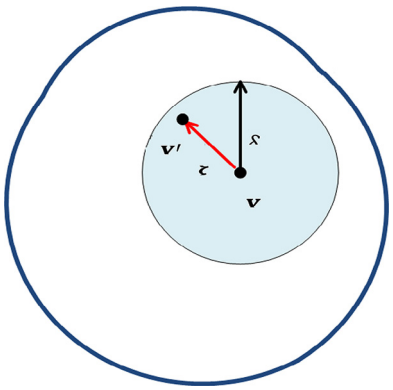

Fig. 1. The peridynamics model.