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Forced vibration of axially moving beam with internal resonance in the supercritical regime

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ABSTRACT

Local and global resonances under the condition of 3:1 internal resonance of a super-critically axially moving beam, subjected to a harmonic exciting force, are investigated in the present work. The governing equation is derived from the generalized Hamilton's principle and discreted into a multiple-degrees-of-freedom system by the Galerkin's method. In the super-critical regime, the axially moving beam becomes a bistable system with two symmetrical non-trivial equilibrium configurations. Based on the transformation around one of them, natural frequencies and the condition of internal resonance are obtained. By employing the method of multiple scales, resonances for first-two modes and harmonics under the condition of internal resonance are discussed analytically. Total displacement at the middle of the beam is composed by them and confirmed by direct numerical method. Internal resonance is found to have a big effect on the phase angle of and the amplitude. Coupling ship between the first-two modes is verified to be produced by the cubic nonlinearity and the 3:1 commensurability together. The effect of moving speed acting on the internal resonance is discussed and an energy transmission region is found. Different with the internal resonance in the sub-critical regime, most of the transferred energy is absorbed by the quadratic nonlinearity in the super-critical regime. The critical excitation of the local response is predicted by the analytical method and certified by simulations. The global response for the primary resonance has two stable focal points. However, the global response for the secondary resonance only has one stable focal point for the non-trivial equilibrium configuration is counteracted.

1. Introduction

The model of axially moving beams can be discovered in many devices in the industry and daily lives. Such as belt driving devices in machines, metal strip in continuously variable transmission (CVT, commonly used in cars), timing belts in engines and band saws. For the importance of it, the axially moving beam has been extensively researched for over half a century.

In 1965, Mote did a pioneering work to investigate the dynamics of the moving beam. He proposed the governing equation and discussed natural frequencies of axially moving continua [1,2]. With a view to the linear restoring force and the geometric nonlinearity, a coupled planar vibrational governing equation was deduced by Thurman and Mote [3]. On the base of it, various characters of the moving beam were discovered. For example, Wickert uncoupled the equation and obtained a nonlinear integro-partial-differential-equation for describing the transverse oscillation [4]. Pellicano and coworkers investigated the weak nonlinear vibration and bifurcation [5,6]. Parametric excitations also can be manifested by this equation [7–10]. Chen and Yang compared the integro-partial-differential model and the partial-differential model by eliminating the high order terms in the coupled equations [11]. Sandilo and van Horssen researched the initial-boundary value problem for an axially moving tensioned beam [12]. Recently, Yang and Zhang investigated nonlinear dynamics of axially moving beam with coupled longitudinal–transversal vibrations [13]. Moreover, Ding and Zu applied these theories into a factual device [14]. Bagdatli and Bilal employed the multi-scales method for discovering free vibrations of axially moving beam under non-ideal conditions [15]. The boundary condition in this document is something between clamped and simply supported boundaries.

However, many transporting systems are moving at a high speed. For example, in paper production, the paper tapes are transported with longitudinal speeds of up to 3000 m/min [16]. While the axial speed exceeds a critical value, the straight equilibrium position becomes unstable and a supercritical bifurcation occurs. The fundamental role of nonlinearity for supercritical systems becomes indispensable. Analytic

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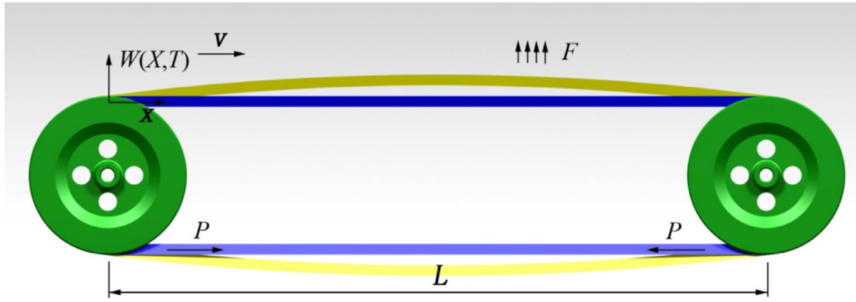

Fig. 1. Diagram of the axially moving beam.

expressions of non-trivial equilibrium configurations were obtained by Wickert [4]. The stability of the non-trivial equilibrium configurations were discussed by Hwang and Perkins [17,18] and they found that only the first one was stable. Based on the stable non-trivial equilibrium configuration, complex responses and stabilities of the flexible structures were investigated [19–22]. Natural frequencies of the super-critically axially moving beam were researched by Ding and Chen [23]. They compared the natural frequencies yielded by the Galerkin method and the differential quadrature method (DQM) and found that the 4-term Galerkin method yielded rather accurate results for the first-two natural frequencies. The steady-state response of axially moving viscoelastic beams under the supercritical traveling speed was investigated by employing the finite difference scheme (FDM) and the differential quadrature scheme respectively [24–27].

For a multiple-degrees-of-freedom system or a continuous model, internal resonance may make the responses for natural modes more complex. Riedel and Tan applied the method of multiple scales, with the longitudinal motion neglected, to research the forced transverse response of an axially moving strip with 3:1 internal resonance [28]. Bifurcations and coupling between natural modes were produced by the internal resonance. This perturbed method was also successfully used to discuss the nonlinear vibrations and 3:1 internal resonance of a tensioned beam on multiple supports [29]. By using Kane's equation, Hu and Feng studied the stability of a slender axially moving beam with internal resonance [30]. In 2005, Sze et al. studied the forced response of an axially moving strip with internal resonance between the first two transverse modes [31] by the incremental harmonic balance method. Ghayesh researched the nonlinear forced dynamics and the bifurcation diagrams with an internal resonance by using the pseudo-arclength continuation technique and the direct time integration [32,33]. In general, all the works demonstrate more complex responses for the interactions of natural modes.

However, the above literatures all focus on the internal resonance of axially moving beams with subcritical speeds. The non-trivial equilibrium configuration has been certified that it can change the nonlinearity of the moving beam for a quadratic nonlinearity is taken into the governing equation. But on the other hand, the 3:1 internal resonance works for the initial cubic nonlinearity. Besides, it was rare before that solvability conditions via the method of multiple scales must be yielded from a third order operator. Consequently, the present study is necessary as a development of all the existing works. In Section 2, a partial differential integral equation of the super-critically axially moving beam is established. Section 3 studies the local steady-state responses of the primary and secondary responses by the method of multiple scales. In Section 4, numerical examples and influences of parameters are discussed in detail. Besides, global responses are investigated by simulating method. At last, some conclusions are presented in Section 5.

2. Mathematical models

Fig. 1 takes a driving belt as an example of the axially moving beam. The axial transmission speed V is considered as a constant. The distance between two belt-wheel tangent points is L, and the tiny change referring to the transverse vibration is ignored. The support at the tangent point is simplified as simply supported. Euler–Bernoulli method is employed here to describe the belt as it is slender. As it is known to all, the drive belt is tensed when it is installed. The initial tension is represented by P here. In addition, only the transverse movement is taken into consideration. Hence, the kinetic energy and the potential energy are

$$T = \int_0^L \frac{1}{2}\rho A \dot{W}^2 \mathrm{d}X = \frac{\rho A}{2}\int_0^L \left(W_{,T} + W_{,X} V\right)^2 \mathrm{d}X,$$
$$V = \int_0^L \left(P\varepsilon_X + \frac{1}{2}A\sigma\varepsilon_X + \frac{1}{2}MW_{,XX}\right)\mathrm{d}X \tag{1}$$

where A is the cross-sectional area, ρ is the density. σ and ε_X denote the disturbance stress and the strain along the beam. M is the bending moment in a micro-segment. Commas preceding T or X denote partial differentiation with respect to T or X, respectively. As the longitudinal oscillation is ignored, the strain along X-axis can be written as

$$\varepsilon_X = \frac{1}{2}{W_{,X}}^2 \tag{2}$$

By employing the generalized Hamilton's principle, the governing equation of the transverse vibration will be deduced.

$$\rho A\ddot{W} + M_{,XX} - PW_{,XX} - A\left(\sigma W_{,X}\right)_{,X} = F \tag{3}$$

where F is an external harmonic force and written as $B\cos(\Omega t)$. As the transverse displacement is much smaller than the length of the belt, the disturbance stress is simplified as a constant value. In addition, the linear part of the Kelvin's material derivative is accurate to describe the constitutive relationship of the belt, which means

$$\sigma = \left(E + \Lambda\frac{\mathrm{d}}{\mathrm{d}T}\right)\varepsilon_X, M = \left(EI + \Lambda I\frac{\mathrm{d}}{\mathrm{d}T}\right)W_{,XX} \tag{4}$$

In Eq. (4), E is the Young's modulus, Λ is the viscoelastic coefficient and I denotes the inertial moment. Consequently, the governing equation Eq. (3) will generate

$$\rho A\left(W_{,TT} + 2W_{,TX}V + V^2 W_{,XX}\right) + \left(EI + \Lambda I\frac{\mathrm{d}}{\mathrm{d}t}\right)W_{,XXXX} =$$
$$+PW_{,XX} + \frac{EA}{2L}W_{,XX}\int_0^L {W_{,X}}^2 \mathrm{d}X + \frac{\Lambda A}{L}W_{,XX}\int_0^L W_{,X}W_{,XT}\mathrm{d}X$$
$$+F \tag{5}$$

Meanwhile, boundary conditions are expressed as

$$W(0,T) = W(L,T) = 0, W_{,XX}(0,T) = W_{,XX}(L,T) = 0 \tag{6}$$

By applying some dimensionless variables and coefficients as follows, Eq. (5) and boundary conditions could be made dimensionless for brevity.

$$w = \frac{W}{L}, x = \frac{X}{L}, t = \frac{T}{L}\sqrt{\frac{P}{\rho A}}, \gamma = V\sqrt{\frac{\rho A}{P}}, b = \frac{BL}{P},$$
$$\omega = \Omega L\sqrt{\frac{\rho A}{P}}, \alpha = \frac{I\Lambda}{L^3\sqrt{\rho A P}}, {k_1}^2 = \frac{EA}{P}, {k_f}^2 = \frac{EI}{PL^2} \tag{7}$$