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On the stability of magnetically levitated rotating rings

Andrea Arena*, Walter Lacarbonara

Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Via Eudossiana 18, Rome 00184, Italy

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ABSTRACT

The dynamic stability of rotating elastic circular rings subject to magnetic levitation and radially recentering magnetic forces is studied. A geometrically exact model of elastic rings deforming in space is formulated in the context of the special Cosserat theory of curved rods. A Lagrangian description of the motion is obtained with respect to the rotating frame. The equations of motion are transformed into a set of ODEs according to a Faedo–Galerkin discretization. The effects of the magnetic and gyroscopic forces on the equilibrium states of the ring are investigated together with the loss of stability of the low-frequency modes. Circular elastic rings with open and closed thin-walled cross sections (i.e., L-shaped and boxed cross sections) are considered. The loss of stability occurring at critical angular speeds where the critical modes become unstable is proved to depend on the ring stiffness and cross-sectional symmetry/asymmetry properties.

1. Introduction

Nonlinear vibrations of thin rings have attracted a lot of attention in the last century. The scientific debate on whether the ring response is softening or hardening has been very lively for decades. Early contributions on the development of mechanical models suitable for the investigation of the nonlinear behavior of ring-like structures were due to Evensen [1–4]. He who showed theoretically and experimentally that the characteristic response of circular rings is softening.

The first efforts toward the determination of the nonlinear response features can be found in the works of Chu [5] and Nowinski [6], where it was determined that the nonlinearity of thin cylindrical shells was of a hardening type. Criticism of these results was brought forth by Evensen who attributed the reason of the hardening response to the incorrect choice of the weight functions adopted in the discretization [1–3], claiming that the weight functions did not correctly satisfy the periodicity condition of the circumferential displacement. In the analytical models employed in previous works the only source of nonlinearity came from the inextensibility condition enforcing the circumferential strain to vanish.

More sophisticated studies were proposed in [5,7,8] in the context of Donnell's or Sander's nonlinear theory of shallow shells whereby the approximate solutions obtained via perturbation techniques proved a softening behavior. A continuum model based on the Timoshenko beam theory accounting for the curved shape through new constitutive laws was presented in [9] where Forgit et al. proposed analytical formulations to determine eigenvalues and eigenfunctions of vibrating rings of arbitrary cross-sectional shape. More recently, a finite element procedure was successfully adopted in [10,11] to obtain the backbone curves of the lowest three driven modes of elastic oval rings which confirmed the characteristic softening behavior predicted by Evensen [4,12]. Most of the cited works dealt with planar rings featuring linearly elastic constitutive laws. In these models, ad hoc mechanical assumptions are often adopted. Such ad hoc models can be inadequate to describe the nonlinear 3D motions exhibited by rings designed for engineering applications involving new composite and multifunctional materials, interactions with magnetic forces or the presence of gyroscopic forces.

Ring-like structures range from large, macro elements, typical for civil, mechanical and aeronautical engineering applications, to micro/nano-mechanical devices, such as the most recent gyroscopes based on ring microresonators [13]. The materials employed in these devices often exhibit nonlinear constitutive behavior [14] which, together with geometric nonlinearities, can severely influence the accuracy of the mechanisms exploiting the micro motions as shown in [15]. In recent works [16,17], an asymptotic approach was adopted to investigate nonlinear vibrations of nonlinearly elastic circular rings. In particular, it was shown that there are thresholds in the nonlinear constitutive laws separating softening from hardening flexural behaviors. Within the broader context of rotating structures, linear and nonlinear vibrations of rotating rings have been investigated for decades. The rich dynamical behavior of rings associated with gyroscopic effects, including the loss of stability at a critical speed and the frequency split of the flexural modes, attracted a lot of interest for the undesirable consequences that such behaviors can have in high precision micro/nano-mechanical devices, such as micro-

* Corresponding author Tel: +390644585885.
E-mail addresses: andrea.arena@uniroma1.it (A. Arena), walter.lacarbonara@uniroma1.it (W. Lacarbonara).

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gyroscopes, or in macroscale structural components such as centrifugal separators or gas turbines [18].

The earliest contribution on rotating rings was due to Bryan in 1890 [19] followed by several authors [20–23] who took inspiration from his work developing linear and nonlinear theories, based on ad hoc kinematic assumptions and boundary conditions, and theoretically validated the phenomenon of separation of the flexural modes frequencies occurring at increasing angular speeds. This characteristic behavior was further validated by the experimental work of Endo et al. [24] providing the frequency measurements of the forward and backward flexural traveling waves. Endo et al. also ascertained that the flexural modes do not become unstable within the whole range of experimentally investigated angular speeds. The unforced planar motion of nonlinearly elastic and viscoelastic rotating rings was studied in [25] where the important role played by the shear deformation was highlighted. An interesting contribution on the effects of the Coriolis forces in elastic rings rotating about an arbitrary axis was presented in [26]. By employing a linear model of elastic rings, a comprehensive study was carried out about the influence of the gyroscopic terms on the modal characteristics and it was shown that spinning rates about axes in the plane of the ring always cause coupling between in-plane and out-of-plane motions. On the other hand, the influence of geometric nonlinearities on modal couplings in rotating rings was investigated in [27]. By employing an approximate nonlinear model, the authors observed the so-called stiffening effect in the frequency separation occurring at increasing speeds. Such a phenomenon was also observed in rotating disks and beams and in early studies of rotating rings on elastic foundations [28]. The most recent contribution on this topic can be found in [29] where small-amplitude oscillations about the undeformed ring state were investigated. The influence of rotational inertia and pre-twisted configurations on the vibration modes was investigated for rings with periodic boundary conditions and for clamped rings.

The present work deals with a fully nonlinear 3D model of unshearable elastic rings rotating about their polar axis and subject to magnetic levitation forces together with radially recentering magnetic forces. This peculiar mechanical system has applications in diverse fields such as high precision machineries and energy and is also envisioned for applications ranging from future shaftless rotors to contactless micro/nanoengines. This is why the study of the ring stability can pave the way to a great wealth of new designs.

A nonlinear continuum model is necessary to correctly investigate the dynamic stability via consistent linearizations as well as nonlinear phenomena that can be exploited for advanced designs. The objectives of this work are multi-fold: (1) present a nonlinear model describing the 3D finite motions of rings including the effects of nonsymmetry of the cross sections (which causes a natural coupling between in-plane and out-of-plane dynamics [30]); (2) discuss interesting phenomena due to the simultaneous presence of nonlinearities arising from levitation and radial magnetic forces; (3) present numerical investigations into the loss of stability of some rigid-like motions observed when the angular speed is increased. To this end, by considering two cross-sectional shapes, the nonsymmetry of the ring cross sections is shown to play a major role on the ring dynamic stability.

2. Mechanical formulation

A circular elastic ring subject to levitation and recentering magnetic forces is assumed to rotate about its polar axis with a prescribed angular speed. The special Cosserat theory of curved rods is employed in the context of a Lagrangian formulation whereby the kinematic parameters are introduced to describe the motions with respect to the rotating frame. The mechanical ring model takes into account sufficiently thin, slender rings for which the shear strains are negligible. The behavior is thus dominated by stretching/flexural/torsional deformations. Moreover, the ring cross section is assumed to be thin-walled and to exhibit eccentricity between the elastic center and the center of mass as is the case with most of the engineering applications making use of rings. To study the vibrational features of this dynamical system and to predict the loss of stability due to the combined effects of the gyroscopic and the motion-dependent magnetic forces, detailed eigenvalue analyses are carried out on the linear equations obtained via linearization of the nonlinear equations of motion about the rotating equilibrium states for a given angular speed.

Kinematics. A fixed right-handed frame $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ is introduced in Euclidean space, with $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ being the plane in which the base curve of the ring lies in its stress-free state (see Fig. 1) and $\boldsymbol{e}_3$ collinear with the ring polar axis. To attain broader generality, the ring is assumed to have open or closed thin-walled cross sections generally lacking symmetry. The cross sections are assumed rigid in their own plane.

The circular base curve, described by the arclength s with origin at some arbitrary point, is assumed as the locus of the elastic centers C^{E} of the ring cross sections. The orientation of the cross sections in the reference (stress-free) configuration is described by the local principal triad $\{\boldsymbol{b}_1^{\circ}(s), \boldsymbol{b}_2^{\circ}(s), \boldsymbol{b}_3^{\circ}(s)\}$. At time t, the current orientation of the ring cross section is described by the orthonormal triad $\{\boldsymbol{b}_1(s,t), \boldsymbol{b}_2(s,t), \boldsymbol{b}_3(s,t)\}$. Such triad is obtained via a finite rotation of the frame $\{\boldsymbol{b}_1^{\circ}(s), \boldsymbol{b}_2^{\circ}(s), \boldsymbol{b}_3^{\circ}(s)\}$ through a sequence of two flexural rotations denoted by $\phi_3(s,t)$ and $\phi_2(s,t)$ and the twisting rotation denoted by $\phi_1(s,t)$, respectively. The resulting orthogonal tensor denoted by $\mathbf{R}(s,t)$ is parametrized in terms of the described rotations in the form given in the Appendix.

In the stress-free configuration, the ring base curve is a circle described by the position vector $\boldsymbol{r}^{\circ}(s) = -r^{\circ}\, \boldsymbol{b}_2^{\circ}$, where r° is the radius of the undeformed ring. Such a curve is characterized by a constant geometric curvature $\boldsymbol{\mu}^{\circ} = \mu^{\circ}\, \boldsymbol{b}_3^{\circ}$ (where $\mu^{\circ} = 1/r^{\circ}$). The displacement of the base line from the reference to the current configuration at time t is described by the vector $\boldsymbol{u}(s,t) = u_1(s,t)\, \boldsymbol{b}_1^{\circ} + u_2(s,t)\, \boldsymbol{b}_2^{\circ} + u_3(s,t)\, \boldsymbol{b}_3^{\circ}$. Thus the current position of the elastic centers C^{E} is given by the vector $\boldsymbol{r}(s,t) = \boldsymbol{r}^{\circ}(s) + \boldsymbol{u}(s,t) = u_1(s,t)\, \boldsymbol{b}_1^{\circ} + \left(u_2(s,t) - r^{\circ}\right) \boldsymbol{b}_2^{\circ} + u_3(s,t)\, \boldsymbol{b}_3^{\circ}$.

The vector of the generalized strain parameters is defined as $\boldsymbol{v}(s,t) = \partial_s \boldsymbol{r}(s,t)$ (the notation ∂_s here and henceforth indicates partial differentiation with respect to the arclength s). The components of $\boldsymbol{v}(s,t)$ in the cross-section-fixed frame can be obtained as $\nu = \boldsymbol{v} \cdot \boldsymbol{b}_1$ (stretch), $\eta_2 = \boldsymbol{v} \cdot \boldsymbol{b}_2$ (shear strain in the $\boldsymbol{b}_2$ direction), $\eta_3 = \boldsymbol{v} \cdot \boldsymbol{b}_3$ (shear strain in the $\boldsymbol{b}_3$ direction), respectively. The dot between two vectors indicates the dot product. The ring deformation modes include also bending and twisting described by the incremental curvature vector $\boldsymbol{\mu}(s,t) = \mu_1\, \boldsymbol{b}_1 + \mu_2\, \boldsymbol{b}_2 + \mu_3\, \boldsymbol{b}_3$ calculated according to $\partial_s \boldsymbol{b}_i = \check{\boldsymbol{\mu}} \times \boldsymbol{b}_i$ where $\times$ represents the cross product and $\check{\boldsymbol{\mu}}$ denotes the total elasto-geometric curvature vector expressed as $\check{\boldsymbol{\mu}}(s,t) = \mathbf{R} \cdot \boldsymbol{\mu}^{\circ} + \boldsymbol{\mu}(s,t)$ (for more details about the justification of this formula, please see Chap 7, Sect. 7.2 of [31]). The components μ_i $(i = 1, 2, 3)$ of the incremental curvature vector are given in the Appendix.

Equations of motion. The generalized stress resultant and moment are expressed as $\boldsymbol{n}(s,t) = N\, \boldsymbol{b}_1 + Q_2\, \boldsymbol{b}_2 + Q_3\, \boldsymbol{b}_3$ and $\boldsymbol{m}(s,t) = T\, \boldsymbol{b}_1 + M_2\, \boldsymbol{b}_2 + M_3\, \boldsymbol{b}_3$, respectively. The generalized stress components N, Q_2 and Q_3 have the meaning of tension and shear forces, respectively, while T, M_2 and M_3 represent the torque and the bending moments, respectively.

For cross sections having the elastic center C^{E} not coinciding with the center of mass C of the ring cross sections, the time rates of change of linear and angular momentum per unit reference length are given by $\partial_t \boldsymbol{l}(s,t) = \rho A\, \partial_{tt} \boldsymbol{r} + \partial_t \tilde{\boldsymbol{\omega}} \times \rho \boldsymbol{S} + \tilde{\boldsymbol{\omega}} \times \left(\tilde{\boldsymbol{\omega}} \times \rho \boldsymbol{S}\right)$ and $\partial_t \boldsymbol{h}(s,t) = \rho \boldsymbol{J} \cdot \partial_t \tilde{\boldsymbol{\omega}} + \tilde{\boldsymbol{\omega}} \times \left(\rho \boldsymbol{J} \cdot \tilde{\boldsymbol{\omega}}\right) + \rho \boldsymbol{S} \times \partial_{tt} \boldsymbol{r}$, respectively. Here and henceforth, ∂_t and ∂_{tt} indicate partial differentiation with respect to time t, ρA is the ring mass per unit reference length, $\rho \boldsymbol{S} = \rho S_2\, \boldsymbol{b}_2 + \rho S_3\, \boldsymbol{b}_3$ is the vector of first mass moments ρS_i of the ring cross section, and $\rho \boldsymbol{J}$ is the positive-definite tensor of second moments of mass whose nontrivial components are the mass moments of inertia ρJ_{11}, ρJ_{22} and ρJ_{33} with respect to $\boldsymbol{b}_1$, $\boldsymbol{b}_1$ and $\boldsymbol{b}_3$, respectively. Furthermore, $\tilde{\boldsymbol{\omega}}$ indicates the angular velocity of the ring cross sections.

By letting $\boldsymbol{f}(s,t)$ and $\boldsymbol{c}(s,t)$ denote the external force and couple per unit reference length, including the dissipative forces, the local state-