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Contact stiffness of multiscale surfaces by truncation analysis

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ABSTRACT

In this paper, we study the contact stiffness of a fractal rough surface compressed by a rigid flat plane. A numerical model based on the analysis of flat punch indentation is proposed for simulated hierarchical surfaces, which are generated using statistical and fractal descriptors collected by surface profilometry. The contact stiffness of surfaces under increasing normal load is determined on the basis of the total truncated area at varying heights. The results are compared with experimental data from nanoindentation on four types of treated rough surfaces, showing good agreement with experimental observations below a certain truncation depth. Furthermore, the limits of the model's validity are discussed by focusing on surface geometries and deformation of contacting asperities. With this proposed truncation method, we present a parametric analysis to establish a correlation between contact stiffness and surface roughness descriptors. The contact stiffness shows a unified power-law scaling with respect to the applied load over a wide range for simulated surfaces with distinct sets of roughness descriptors. The exponent of the power-law relationship is found to correlate positively to the fractal dimension while its amplitude is inversely correlated to the surface roughness amplitude. This study provides an easily implemented and computationally efficient method to connect mechanical behaviour with multi-scale surface structure, which can be utilized in design and optimization of engineering applications involving rough contacts.

1. Introduction

The morphology of surfaces plays a determining role in interfacial phenomena including friction, adhesion, sealing, lubrication and thermal and electrical conductance [1,2]. When two solids with rough surfaces are squeezed together, the area of true contact consists of numerous contact patches of various sizes and generally comprises only a small fraction of the nominal contact area. Over the years, numerous divergent approaches have been developed to explore load dependent contact behaviour at rough surfaces, yet the scientific community remains divided regarding the reliability of their predictions [1,3–6]. The various asperity-based approaches to contact models reported in the literature can be categorized as: (1) multi-asperity contact models (i.e., statistical models), in which the heights and / or curvatures of asperities follow given statistical distributions [7–10]; and (2) surface fractality models, including multi-scale models [11–17], the boundary element method (BEM) [6] and Persson's theory [18, 19]. Further to this binary classification, the comparison between various reported models reveals discrepancies with noticeable differences between each other and with experimental results [17,19–21].

Within the above-mentioned approaches to surface structure simulation, it is also necessary to consider the contact behaviour of an individual asperity ranging from elastic, through elasto-plastic, to fully plastic deformation [8,22–29]. For purely elastic or plastic contact, the classic Hertzian model [30] and fully plastic model [31] can be applied, respectively. However, for elastoplastic regimes, individual asperity models may yield different contact responses, dominated by different deformation mechanisms [32,33]. Particularly, a self-consistent analysis was put forward by Storåkers, et al. [34] considering the general visco-elasto-plastic material indented by a spherical object. Additionally, shoulder-to-shoulder contact models for misaligned asperities were introduced to include oblique contact between pair asperities [15,35,36].

The contact behaviour of individual asperities can be combined to shed light on overall system behaviour by considering statistical and/or fractal approaches to describe surface morphologies. Pioneered by Greenwood and Williamson [7], multi-asperity contact models are based on the statistical height distribution (Gaussian or non-Gaussian), while assuming that the deformation of a given asperity is not influenced by that of neighbouring asperities. Within this framework, various implementations, considering different asperity geometries, have been developed over the past decades for the analysis of individual asperity deformation. In practice, statistical parameters for characterizing surface topography, such as variance of heights, slope, curvature, etc., have been used in this class of contact models. But these implementations assume features at a given narrow range of scales and thus depend on the

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resolution of the surface measuring apparatus and sample length [13]. However, most natural surfaces exhibit features across a wide range of length scales [37], involving diverse morphologies, which bring about complexities in the modelling of interfacial properties [3,38]. In addition, these statistical models were constructed by assuming that micro-contact forces arise principally due to deformation of asperities and are calculated considering only the base wavelength or a certain range of wavelengths, characterized by the roll-off and cut-off wavevectors of the power spectra of rough surfaces [4,12,17,19,39]. This assumption neglects the contribution of fine geometries at small length scales to the contact properties. In the fractal-theory related models, the finer surface features have been included in terms of overall contact area using an integration process [11,12,28,40].

An early appreciation for the significance of the multi-scale nature of surfaces was demonstrated by Archard [41]. In the contact theory using scale-independent parameters, Majumdar and Bhushan [42] utilized the fractal theory of Mandelbrot [43] to describe the distribution of contact areas between two rough surfaces. Yan and Komvopoulos [12] extended this theory to contact problems of three-dimensional rough surfaces, revealing the variations of the contact force and real contact area during quasi-static loading. Ciavarella, et al. [44] developed a two-dimensional fractal-based model for sinusoidal elastoplastic surfaces, and expanded the analysis to provide contact stiffness and interfacial resistance. Further relevant work was conducted by Jackson and Streator [13] using three-dimensional sinusoidal surfaces, considering the frequency spectrum of the surfaces. Pohrt and Popov [6] deduced an empirical contact stiffness model by means of the boundary element method (BEM). The validity of the method of reduction of dimensionality, by which three-dimensional contact problems are mapped onto one-dimensional elastic contacts, has been investigated for non-adhesive contact of any axisymmetric bodies [45–47]. However, the fractal dimension has been shown to change due to an applied load [38,48,49] and thus an assumption of invariant fractal dimension in fractal-theory related models may only be reasonable within a certain range of loading. Moreover, various methods [20,50,51] can be applied to quantify the fractality of a rough surface [19,20,52,53] possibly resulting in different values of the obtained fractal dimension, thus the adaptation of these methods merits further study to provide a consistent result in contact mechanics models.

The true area of contact formed between two surfaces is of prime interest and is governed by morphology, material properties, and loading conditions. The contact properties of rough surfaces subjected to an applied normal load can be, at some level, interpreted by considering the true interfacial contact area along with surface descriptors [4,46]. However, determining the real contact area between contacting bodies through experimental or numerical analyses remains challenging. Factors that influence true contact area include the asperity height, curvature, the Poisson's ratio of the material, strength and hardness and the existence of superimposed smaller asperities with similar or divergent scale-dependent properties [54]. With time, the sliding induced by expanding contacting spots [55] further affects contact morphology. Moreover, most natural surfaces exhibit features across a wide range of length scales the extent of which depends on simulation or measurement resolution [56,57].

Despite the fact that the contact area tends to present scale-dependent properties with contacting asperities in the status of incomplete contact (with non-contacting zones surrounded by the contacting ones), the truncated areas at various depths for a given resolution can be employed to represent the real contact area. This was first proposed by Abbot and Firestone [31] to describe a wear process rather than indentation or flattening. Along this line, proposed truncation models [58,59] have assumed that the contact area of an asperity pressed against a rigid flat can be approximately calculated by mathematically truncating the asperity tip. This is a reasonable assumption for small to medium interferences as within this range asperities tend to plastically deform due to the small radius of curvature. The average pressure between an asperity and a flat punch can simply be assumed to be equal to hardness, or can be related to material yield strength [7,26]. However, Jackson and Green [22] showed that this simplification results in an inverse hardening process in which the hardness actually decreases with increasing interference.

In addition to contact area the present work focuses on interfacial contact stiffness. An understanding of contact stiffness is important in contact mechanics, which plays a central role in governing the stress-dependent electrical and thermal transport between two contacting solids [6,60,61]. In the past decade, the relationship between surface structure and interfacial stiffness has been intensively studied numerically and experimentally. Numerical analyses, using methods of molecular dynamics, finite element analysis, etc., generally confirm linear proportionality between normal force and contact stiffness [39,62,63], as supported by the Greenwood–Williamson model [7] and Persson's theory [19,64]. However, other studies have reported that, for small to medium loads, the logarithm of stiffness exhibits close proportionality to the logarithm of the applied normal force [6,45,52]. In other words, the contact stiffness, k, is a power function of the normal force, F_N, as $k \propto F_N^{\alpha}$ (with $\alpha < 1$), which differs from the work mentioned above. Numerous experimental studies have been carried out on rough surfaces to ascertain the relationship between surface structure and interfacial behaviour under load using diverse experimental approaches and materials. Jiang, et al. [52] measured the normal contact stiffness of cast iron specimens produced using different machining methods. Wang, et al. [19] measured the contact stiffness of a rubber block squeezed against different concrete and asphalt road surfaces. Buczkowski, et al. [17] compared the normal contact stiffness determined using ultrasonic measurements with the fractal model based on Weierstrass–Mandelbrot function. Zhai, et al. [20] recently evaluated the contact stiffness at aluminium surfaces by nanoindentation tests utilizing different sized flat tips to achieve a wide range of applied stress levels. These experimental studies support the power law relationship between the contact stiffness and the applied load for certain stress ranges.

The main purpose of the paper is to propose a comprehensive contact analysis method for a three-dimensional rough surface compressed by a rigid flat. The proposed truncation method is applied to simulated fractal surface structures characterized using various statistical and fractal descriptors, to interpret the variation of contact stiffness under increasing normal loading using experimental results for reference. Iteration procedures employed in simulating fractal rough surfaces ensure a description with identical parameters which are critical in determining the normal contact stiffness. The applicability and repeatability of the presented method is discussed based on geometrical features and analyses of the studied rough surfaces. This study provides an easily-incorporated and highly-effective numerical method for predicting contact stiffness under conditions of small to medium loads. Following validation, a parametric analysis is conducted for simulated surfaces varying in surface topologies, allowing the obtained contact stiffness to be related to surface roughness descriptors. Finally, correlations between normal contact stiffness and roughness descriptors have been established and discussed with respect to fractal dimension values obtained using different methods.

2. Theoretical framework

The deformation mechanics of contacting surface asperities remains a topic of significant debate in the research community. Under most applied conditions the true contact area between rough surfaces involves only a small fraction of the nominal contact area, and for this reason the total real contact area can be considered to approximately equal to the truncation area in the present method. We consider a simple extension of the contact analysis for indentation by a flat punch [65], which reveals that there is a relation between contact stiffness, contact area, and elastic