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On nonlocal integral models for elastic nano-beams

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Abstract

Nonlocal integral constitutive laws, for application to nano-beams, are investigated in a general setting. Both purely nonlocal and mixture models involving convolutions with averaging kernels are taken into account. Evidence of boundary effects is enlightened by theoretical analysis and numerical computations. Proposed compensation procedures are analyzed, relevant new results are evidenced and confirmed by computations. The strain-driven model and related local-nonlocal mixtures are addressed, with singular phenomena foreseen and numerically quantified. Effectiveness of the recently proposed stress-driven nonlocal elastic model is discussed and illustrated by description of a general solution procedure for nonlocal elastic beams. Comparisons between strain-driven models, stress-driven models and local/nonlocal mixtures are considered from theoretical and computational perspectives. Examples of statically determinate and indeterminate beams are elaborated to show that an effective simulation of scale effects in nano-structures, ensuring existence and uniqueness of solution for any data, is provided by the stress-driven model.

Key words: Nonlocal elasticity, Integral elastic law, Boundary effects, Local/nonlocal mixtures, Nano-beams, CNT

1. Introduction

Nano-structures such as carbon nanotubes (CNTs) exhibit size effects, whose evaluation is conveniently simulated by a continuum mechanics approach in which nonlocal constitutive models are adopted.

In a paper on screw dislocations and Rayleigh surface waves [1] Eringen was the first to introduce strain-driven nonlocal elastic laws in which the stress field was expressed by convolution of the local elastic stress with averaging kernels consisting in fundamental solutions of differential problems.

In dealing with unbounded domains, integral convolutions with smoothing kernels were replaced with equivalent differential equations with boundary condition of vanishing at infinity.

Eringen's differential nonlocal elastic equations were improperly later applied in [2] for investigating size effects in bounded nano-beams. Accordingly, in modelling cantilevers under end-point loading, used as actuators in nanotechnology, paradoxical results were detected in [3, 4].

This notwithstanding, differential formulations were thenceforth adopted as reference constitutive schemes in simulating the structural behaviour of devices at nanoscale. Modifications of the differential formulation were examined in [5, 6].

It is to be underlined that, on a bounded interval, the integral convolution problem, involved in Eringen's integral constitutive law with the bi-exponential kernel, implies the fulfilment of homogeneous boundary conditions [7].

What is more, equivalence between the integral constitutive law and the differential equation holds if and only if corresponding constitutive boundary conditions (see Eq.(9)) are imposed. These topics were first addressed in [8] with reference to extensional behaviour of nano-bars.

A list of recent contributions can be found in [9].

A definite explanation of paradoxical result has been finally contributed in [10] with reference to flexural behaviour of nano-beams. The conclusion was that the strain-driven nonlocal integral elastic law and equilibrium requirements on the bending field are mutually incompatible for structural models of engineering interest, thus leading to formulation of unsolvable elastostatic problems.

To overcome ill-posedness of strain-driven nonlocal elastic problems, a mixture of local-nonlocal elasticity was adopted in [11, 8, 12, 13, 14, 15] on the basis of the original proposal by Eringen in [16, 17, 18].

Proposals originally made in [19, 20, 21] with reference to nonlocal damage mechanics, were applied to nonlocal elasticity of nano-beams to compensate for boundary layer effects and adopted in numerical computations [22].

Inconsistencies in nonlocal structural problems, formulated according to the strain-driven integral elastic law, can be bypassed by the proposal of a new nonlocal stress-driven integral elastic relation contributed in [23, 24].

As foreseen by a general reasoning, in applying the stress-driven model to bending of nano-beams and stretching of nano-bars, all shortcomings, inherent to nonlocal models defined by a strain-driven integral convolution, are eliminated from the root.

The distinctive characteristic of the new theory consists in setting the stress field as input of the integral nonlocal law, the nonlocal elastic strain being the output.

A comparison with Eringen's strain-driven nonlocal law, reveals that input and output are swapped, the two integral laws being not one the inverse of the other.

This feature is decisive since, in elastostatic problems based