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Frictional contact involving a multiferroic thin film subjected to surface magnetoelectroelastic effects

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ABSTRACT

This paper reports a size-dependent semi-analytical model (SAM) on the frictional magnetoelectroelastic (MEE) contact of a multiferroic thin film and a rigid insulating sphere subjected to surface MEE effects. The Huang-Yu and Gurtin–Murdoch theories are extended to account for the effects of surface MEE. The frequency response functions (FRFs) for the MEE film are analytically derived, which incorporate the surface effects and are consequently converted to the corresponding influence coefficients (ICs). The conjugate gradient method (CGM) is used to obtain the unknown pressure distribution and the fast Fourier transform (FFT) is involved to calculate the surface electric/magnetic potentials and subsurface stresses. The proposed model is implemented to analyze the influences of surface MEE, multiferroic film thickness, and friction coefficient on the contact behaviors, including pressure/stresses and electric/magnetic potentials. A sensitivity analysis is conducted to evaluate the influence of surface parameters and their coupling on the contact behaviors. A set of behavior maps for the pressure and electric/magnetic potentials are constructed to reveal the influences of film thickness and material characteristic length, which can be used to determine the appropriateness of the geometry configuration (i.e. film or half-space) and surface behaviors (i.e. with or without considering the surface MEE effects).

1. Introduction

Multiferroic materials are noteworthy for their uniqueness and strong coupling of ferroelectricity, ferromagnetism, and ferroelasticity [1]. These materials, in the form of a thin film bonded to a substrate, are being considered for applications in magnetostrictive sensing, ultrasensitive magnetometry, energy conversion, and electronic instrumentation. Accordingly, a large amount of experimental studies have been performed to explore methods for the syntheses and characterizations of multiferroic films. Park et al. [2] pointed out that the measured results of considered materials at some size range deviated from the expected behaviors. They indicated the lack of fundamental understanding of the size-dependence nature of the magnetic responses and ferroelectric behaviors. Chu et al. [3] revealed a critical film thickness above which the influence of substrate can be neglected. Bluhm et al. [4] revealed that friction coefficient in the ferroelectric domain subjected to a lateral force microscopy (LFM) tip was responsible for the surface electric potential that was dependent on the microstructure of the material surface. Key questions are raised from these experimental observations: (a) What is the limiting dimension above which the nano/micro-structural size effect becomes significant and what difference can the size effect bring about? (b) How does the film thickness affect the magnetoelectroelastic (MEE) responses and is there a general trend for the film thickness effect? (c) What is the general role of friction in coupling the MEE effects in a contact process?

A theoretical method that can precisely describe the features of multiferroic materials is highly desirable for in-depth understanding of their overall responses to contact loading. Light weight and compactification of MEE structures into a thin film would create a high surface-to-volume ratio, which could result in a significant influence of the surface energy on the MEE behaviors of multiferroic nanostructures. As a consequence, classical MEE models, in which the surface effects were largely ignored, would be no longer applicable for investigating the size-dependent behavior of the thin-film materials. Gurtin and Murdoch [5,6] developed a modified continuum-based method to capture surface elasticity effects on the elastic responses of nanostructures, in which the considered solid was described as a bulk bonded with a surface layer of zero thickness. The constitutive laws for the bulk were still those in the classical continuum theories for conventional materials, although the residual

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Nomenclature

a	contact radius, m
a_0	contact radius in multiferroic half-space, m
A_k, $\bar{A}_k$	unknowns in potential functions
B_i	magnetic induction along the x_i direction, N/(Am)
B_α^s	surface magnetic induction, N/A
B_α^0	residual surface magnetic induction, N/A
c_{ijkl}	elastic stiffness tensor, 10^9 N/m^2
$c_{\alpha\beta\gamma\delta}^s$	surface elastic stiffness tensor, 10^9 N/m
C^u, C^φ, C^ψ, C	influence coefficients
d_{kij}	piezomagnetic stiffness tensor, N/(Am)
$d_{\gamma\alpha\beta}^s$	surface piezomagnetic stiffness tensor, N/A
D_i	electric displacement, C/m^2
D_α^s	surface electric displacement, C/m
D_α^0	residual surface electric displacement, C/m
e_{kij}	piezoelectric stiffness tensor, C/m^2
$e_{\gamma\alpha\beta}^s$	surface piezoelectric stiffness tensor, C/m
E_α^s	surface electric field, V/m
f	selected material constant
f^0	f ‘s reference value
g	gap between two surfaces, m
g_{ij}	electromagnetic coefficient, C/(Am)
$g_{\alpha\beta}^s$	surface electromagnetic coefficient, C/A
G^u,G^φ, G^ψ, G	frequency response functions
h_0	initial separation between two surfaces, m
h_t	film thickness, m
H_α^s	surface magnetic field, A/m
i	imaginary unit
l_s	characteristic length, m
m, n	Fourier-transformed variables with respect to x_1, x_2 directions, respectively
m_α	surface bases
n_i	unit normal vectors
M, N	mesh numbers along the x_1, x_2 directions
M_e, N_e	refined mesh numbers
q	shear tractions parallel to the x_1 direction, Pa
Q	applied tangential force along the x_1 direction, N
p	pressure, Pa
P	applied normal force, N
R	radius of a spherical punch, m
s_j	variables relating to material properties
u_i	displacements along the x_i direction, m
V	contact behavior for u_3, u_1, ϕ, φ
x_j	Cartesian coordinates in the spatial domain
Y	shape function

Greek letters

α	distance of a node, (m, n), measured from origin of the frequency domain
γ	refinement level
$\gamma_{\alpha\beta}^s$	surface strain
$\kappa_{\alpha\beta}$	surface curvature tensor with $\kappa_{\alpha\beta} = -u_{i,\alpha\beta}$
δ_{ij}	Kronecker delta function
ε_{ij}	dielectric permittivity, 10^{-9} C^2/(Nm^2)
$\varepsilon_{\alpha\beta}^s$	surface dielectric permittivity, 10^{-9} C^2/(Nm)
μ_{ij}	magnetic permeability, 10^{-6} Ns^2/C^2
$\mu_{\alpha\beta}^s$	surface magnetic permeability, 10^{-6} Ns^{2}m/C^2
μ	friction coefficient
σ_{ij}	stress components, Pa
σ_s	von Mises stress, Pa
$\sigma_{\alpha\beta}^s$	surface stress, N/m
$\sigma_{\alpha\beta}^0$	residual surface tension, N/m
ϕ	electric potential, V
φ	magnetic potential, A
Γ_c	contact zone
Γ	surface energy density
Δ_{xi}	grid size in the x_i directions, m

Special marks

$\approx$	double continuous Fourier transform
ˆ	discrete Fourier transform
$\lvert\cdot\rvert$	determinant of a matrix
IFFT	inverse fast Fourier transform

surface tension and surface elasticity were taken into consideration in the surface layer. It has long been recognized that the size-dependent elastic behaviors of nanostructures can be accurately described by the Gurtin–Murdoch model, whose results well agree with those from experiments and atomistic simulation [7,8]. Huang and Yu [9] presented a surface piezoelectric model, which was a natural extension of the Gurtin-Murdoch model for elasticity, to explain the surface piezoelectric effects on the electroelastic behavior of nanostructured piezoelectric rings. Likewise, surface models have been introduced to capture the surface piezoelectric or MEE effects in a wide variety of nanostructures, such as piezoelectric plates [10], piezoelectric nanotube [11], and MEE plates [12,13].

Implementation of multiferroic thin films to detection and energy conversion systems faces an issue of frictional contact. When the thin film is in contact with another material alike or not, novel phenomena may be expected as a result of the interaction of MEE coupling and surface effects. A number of analytical and numerical works have been committed to investigate some aspects of the contact behaviors of MEE materials. Several examples for the analytical approaches are the work by Hou et al. [14] on the elliptical frictionless contact of MEE half-spaces, the set of exact solutions by Chen et al. [15] for the frictionless contact of a half-space and rigid conductive or insulating indenters, and the study by Zhou and Lee [16] about the frictional sliding contact between a half-plane and a rigid punch. Among the numerical approaches are the modeling by Michopoulos et al. [17] to study the MEE behaviors of multiferrioc materials based on the finite element method (FEM) and the investigation of Rodriguez–Tembleque et al. [18] on the frictional contact of an MEE half-space subjected to a rigid sphere using the boundary element method (BEM). However, little has been done to MEE thin films, although these finite films, rather than a half-space, are the majority in the applications of the multiferrioc materials. Pauk and Woźniak [19] stated that “In many cases the modeling of components of real frictional couples by a half-space is impossible. Many pairs can be often considered as a film’; Chen and Chen [20] pointed out “Modeling of a finite film can be helpful for the design of surfaces with strong wear resistance and novel materials for real applications”. Generally, many studies have been committed to investigate the indentation responses of piezoelectric materials, among them are the solutions by analytical approaches reported in [21,22,23], and those by using the FEM reported in [24] and BEM reported in [25]. However the complexity of MEE coupling could make the frictional contact problems of multiferroic film so complicated that exact analytical solutions can hardly be obtained even if without considering the surface effects, a reasonable approach would be semi-analytical modeling (SAM) built upon core analytical solutions and supported by fast numerical solution approaches. Developing an efficient model for tackling the combined influences of surface effects, film thickness, and sliding friction on the MEE responses of multiferroic thin films, together with a set of effective solution methods, constructs the primary goal of the present work.

SAM has been proven to be efficient for solving both frictionless and frictional contact problems involving half-space [26–28] or layered materials [29–32]. The core analytical solutions to displacements, electric/magnetic potentials, stresses, electric displacements, and magnetic inductions of the MEE thin film should be derived first pertaining to a