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Simulation of a crack in stiffened plates via a meshless formulation and FSDT

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ABSTRACT

A meshless formulation for fracture analysis of stiffened plates is introduced in this paper. Based on an improved meshless model of stiffened plates proposed by the authors, in which flat plate and ribs are combined by implementing the displacement compatibility conditions between them, a crack is introduced by the diffraction method. The expanded basis function and the weight function based on t^{-1} distribution are employed. Inheriting the meshless advantages from the model, the ribs in our formulation can be set at any location on the flat plate, and the remeshing of the flat plate is naturally avoided when rib location changes. Some numerical examples are investigated by the proposed formulation and the commercial FEM software ANSYS, and the accuracy of the proposed formulation is verified. The effects of ribs on the displacement, stress and SIFs of the stiffened plates that have cracks are discussed.

1. Introduction

Because of high strength/weight ratio, stiffened plates have been extensively found in civil and oceanographic engineering, transportation and aerospace industry, etc. Analyzing stiffened plates is no doubt more complicated than analyzing flat plates because of the presence of ribs/stiffeners.

The early researchers used an orthotropic model [1], which approximated stiffened plates with flat plates of equal thickness. Another option was a grillage model [2]. At present, researchers tend to first consider the ribs and flat plate of a stiffened plate separately, and then to combine them together by introducing the displacement coordination between them.

In 1973, Tvergaard [3] investigated local and general buckling of wide panel stiffened by eccentric ribs under compression. Smith [4] studied a ship hull under local compression, and obtained the ultimate longitudinal strength of the hull. Taking plate/rib interplay and welding residual stress into account, an analytical method to study elastic local buckling of a stiffened plate under uniaxial compression was presented by Fujikubo and Yao [5]. Duc et al. [6] carried out a nonlinear dynamic analysis on piezoelectric imperfect stiffened FGM plate. Dang and Kapania [7] presented a Ritz approach for buckling prediction of cracked-stiffened structures. Milazzo and Oliveri [8] analyzed post-buckling of cracked multilayered composite plates through pb-2 Rayleigh–Ritz method, which was followed by their further study on buckling and post-buckling of stiffened composite panels with Ritz approach [9].

For the past decades, numerical tools for stiffened plates developed rapidly, and the finite difference method [10, 11], the energy based approach [12], the finite element method [13–17], BEM-FEM method [18], the compound strip method [19] and the reproducing kernel particle method [20] for stiffened plates were proposed. An Apb-2 Rayleigh–Ritz approach for the dynamic analyses of stiffened plates was proposed by Liew et al. [21] and Xiang et al. [22]. Modeling a plate with an Allman's triangular element and ribs with the Timoshenko beam theory, Nguyen-Minh et al. extended a cell-based smoothed method (CS-FEM-DSG3) for the flexure and dynamic analyses of stiffened folded plates [23]. Duc and coworkers [24–28] made contribution to the research of FGM plates and stiffened FGM shells resting on elastic foundation. Duc et al. [29] simulated dynamic crack propagation in functionally graded glass-filled epoxy. Tinh et al. [30] introduced a meshfree analysis for Reissner–Mindlin plates. A meshless model for the solution of static, free vibration and stability problems of stiffened plates and corrugated plates was established by the authors in [31–34].

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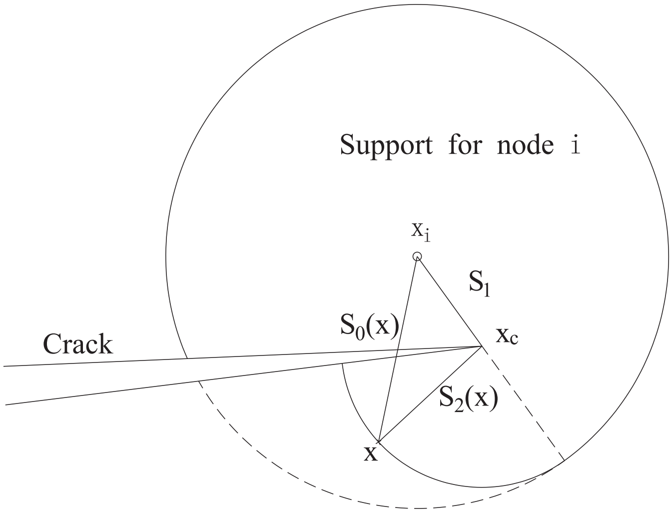

Fig. 1. The DOI of Node i modified by the diffraction method.

In the analysis of fracture problems, traditional FEMs encounter difficulties in handling the movable discontinuities that are not coincident with the initial finite element mesh. *Re*-meshing is inevitable during the entire process of problem evolution, which is very complicated and may lead to accuracy decline. As alternatives, the meshless methods and the extended finite element methods (XFEM) for cracked structures had been developed. The XFEM [35–37] simulates cracks that are not coincident with the finite element mesh, and the method makes the modeling of growing cracks much easier, as no remeshing of the problem domain is needed. Belytschko and coworkers [38–42] developed an Element-free Galerkin (EFG) method for the static and dynamic problem of crack propagation in linear material. The method was able to simulate crack propagation in arbitrary path, and it was applicable to anisotropy and nonlinear material. Xu and Saigal [43, 44] studied type I quasi-static crack propagation of elastic-plastic hardening and non-hardening material with EFG. With a boundary element formulation, Wen et al. [45] analyzed the shear deformable stiffened cracked plates.

In this paper, a meshless formulation based on the first-order shear deformation theory (FSDT) [46] and moving-least square (MLS) approximation [47] is proposed for the simulation of stiffened plate with an edge crack. An improved meshfree model for stiffened plate is introduced, and the edge cracks are involved in the model with the diffraction method [41]. A few examples are analyzed to test the validity of the formulation. The results from FEM analysis given by ANSYS or other researchers are also listed for comparison. The effect of ribs is also discussed based on the proposed formulation.

2. Meshless model for stiffened plate with crack

2.1. The moving-least approximation

According to the MLS [47], $u(x)$ in a problem domain Ω may be approximated by function $u^h(x)$ in sub-domain Ω_x, and

$$u^h(x) = \sum_{i=1}^{m} p_i(x) b_i(x) = \boldsymbol{p}^T(x)\boldsymbol{b}(x), \tag{1}$$

where h defines the domain of influence (DOI) of the nodes, $p_i(x)$ are basis functions, m is their number, and $b_i(x)$ are unknown coefficients. In this formulation, the quadratic basis

$$\boldsymbol{p}^T = [1, x, x^2] (m = 3) \tag{2}$$

is used for the ribs.

Stiffened plates with edge cracks will be studied in the paper, and therefore a discontinuous problem will be involved. In order to introduce the singularity of $r^{1/2}$ at the crack tip and to give better solution, the expanded basis function

$$\boldsymbol{p}^{\mathbf{T}} = [1, x, y, \sqrt{r}]\,(m = 4) \tag{3}$$

is employed for the plate, where r represents the polar coordinate whose origin is at the crack tip.

From Eq. (1), the coefficients $b_i(x)$ are obtained from a L_2 norm

$$L = \sum_{i=1}^{\bar{N}} \omega_i(x)[u^h(x) - u_i]^2 = \sum_{i=1}^{\bar{N}} \omega_i(x)[\boldsymbol{p}(x_i)^T \boldsymbol{b}(x) - u_i]^2, \tag{4}$$

Where $\bar{N}$ defines the number of nodes in sub-domain or DOI Ω_x, $\omega_i(\boldsymbol{x}) = \omega(\boldsymbol{x} - \boldsymbol{x}_i)$ is the weight function for Node i, and u_i the nodal parameters.

$\frac{\partial L}{\partial \boldsymbol{b}(x)} = 0$ results in

$$\boldsymbol{b}(x) = \mathbf{H}^{-1}(x)\mathbf{R}(x)\boldsymbol{u}, \tag{5}$$

where

$$\mathbf{R}(x) = [\omega_1(x)\boldsymbol{p}(x_1) \quad \omega_2(x)\boldsymbol{p}(x_2) \quad \cdots \quad \omega_{\bar{N}}(x)\boldsymbol{p}(x_{\bar{N}})], \tag{6}$$

$$\mathbf{H}(x) = \sum_{i=1}^{\bar{N}} \omega_i(x)\boldsymbol{p}(x_i)\boldsymbol{p}^T(x_i). \tag{7}$$

Substituting Eq. (5) into Eq. (1), we have

$$u^h(x) = \sum_{i=1}^{\bar{N}} N_i(x) u_i, \tag{8}$$

where

$$N_i(x) = \boldsymbol{p}^{\mathbf{T}}(x)\mathbf{H}^{-1}(x)\mathbf{R}_i(x) \tag{9}$$

are the shape functions.

2.2. Modification of the weight function

In this paper, we take a cubic spline function as the aforementioned weight function:

$$\omega(d_i) = \begin{cases} \frac{2}{3} - 4d_i^2 + 4d_i^3, & d_i \le \frac{1}{2} \\ \frac{4}{3} - 4d_i + 4d_i^2 - \frac{4}{3}d_i^3, & \frac{1}{2} < d_i \le 1 \\ 0, & d_i > 1 \end{cases} \tag{10}$$

where d_i is the range from Node i to the evaluation point $\boldsymbol{x}$. However, in order to introduce a crack in the DOI of a node, the diffraction method [41] was used to modify d_i so that the DOI of a node wraps around the crack tip (Fig. 1), which is similar to light diffraction:

$$d_i = \left(\frac{s_1 + s_2(x)}{s_0(x)}\right)^{\lambda} s_0(x), 1 \le \lambda \le 2 \tag{11}$$

where

$$s_0(\boldsymbol{x}) = \|\boldsymbol{x} - \boldsymbol{x}_i\|, s_1(\boldsymbol{x}) = \|\boldsymbol{x}_C - \boldsymbol{x}_i\|, s_2(\boldsymbol{x}) = \|\boldsymbol{x} - \boldsymbol{x}_C\|,$$

and $\boldsymbol{x}_c$ is the crack tip coordinates. We substitute Eq. (11) into Eq. (10) and obtain the weight function. The derivatives are computed under chain rule:

$$\frac{\mathrm{d}\omega}{\mathrm{d}x} = \frac{\partial \omega}{\partial d_i}\frac{\partial d_i}{\partial x} \tag{12}$$

2.3. Displacement approximation

The meshless model of a stiffened plate (Fig. 2) consists of two ribs (considered as beams) and a flat plate. The beams and the flat plate are discretized by a number of nodes. The degree of freedom (DOF) for a node of the flat plate is defined as $(u_p v_p w_p \varphi_{px} \varphi_{py})$, and DOF for a node of the x-stiffener is $(u_s w_s \varphi_s)$. We ignore the torsional stiffness and in-plane bending of the stiffener. For convenience, only x-stiffener appears in our derivation, and y-stiffeners can be added to the derivation likewise.