Discrete and nonlocal models of Engesser and Haringx elastica

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Abstract

In this paper a generalized discrete elastica including both bending and shear elastic interactions is developed and its possible link with nonlocal beam continua is revealed. This lattice system can be viewed as the generalization of the Hencky bar-chain model, which can be retrieved in the case of infinite shear stiffness. The shear contribution in the discrete elastica is introduced by following the approach of Engesser (normal and shear forces are aligned with and perpendicular to the link axis, respectively) and that of Haringx (shear force is parallel to end section of links), both supported by physical arguments. The nonlinear analysis of the shearable-bendable discrete elastica under axial load is accomplished. Buckling and post-buckling of the lattice systems are analysed in a geometrically exact framework. The buckling loads of both the discrete Engesser and Haringx elastica are analytically calculated, and the post-buckling behavior is numerically studied for large displacement. Nonlocal Timoshenko-type beam models, including both bending and shear stiffness, are then built from the continualization of the discrete systems. Analytical solutions for the fundamental buckling loads of the nonlocal Engesser and Haringx elastica models are given, and their first post-buckling paths are numerically computed and compared to those of the discrete Engesser and Haringx elastica. It is shown that the nonlocal Timoshenko-type beam models efficiently capture the scale effects associated with the shearable-bendable discrete elastica.

Keywords: Lattice; Discrete elastica; Shear effect; Nonlocal beam mechanics; Timoshenko beam elements; Scale effect; Buckling; Bifurcation; Post-buckling; Finite Difference Methods

1. Introduction

In this paper the possible link between discrete beam systems and their continuum analogies for structural mechanics applications is investigated. One related question is whether we can build nonlocal beam models from discrete systems composed of rigid elements linked by concentrated or distributed interaction. A pioneer model for relating discrete beam mechanics with its continuum analogue is the so-called *Hencky*